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by

David Paul Davison, Jr.

December 1988

Thesis Advisor: D. L. Walters

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This thesis provides a preliminary investigation of a calibration process

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An Investigation Into Backscattered Cross Section Calibration of an Acoustic Sounder Used for Analysis of Lower Atmospheric Turbulence

by

David Paul Davison, Jr.
Lieutenant, United States Navy
B.S., Texas Agricultural and Mechanical University, 1980

Submitted in partial fulfillment of the
requirements for the degree of

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Atmospheric temperature structure fluctuations quantified by C_T^2 degrade the spatial and temporal coherence of electromagnetic and acoustic waves propagating in the atmosphere. A computer controlled atmospheric echosounder developed at the Naval Postgraduate School measures a time averaged C_T^2 profile of the lower atmosphere. Assigning the proper C_T^2 values to the backscattered return signals depends on an accurate calibration of this instrument. Calibration involves determination of the product of the echosounder's transmission efficiency E_t and reception efficiency E_r.

This thesis provides a preliminary investigation of a calibration process using pulsed acoustic energy backscattered from hard spheres. Supporting software calculates the desired product E_rE_t based on an assumption of echosounder efficiency reciprocity. Results of the calibration process investigation indicate this assumption may be invalid. The results also indicate the software performs as intended and that the proposed calibration method possesses sufficient merit to warrant further development.

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I. INTRODUCTION

The variational effect of wind shears, convection, and temperature gradients on atmospheric density distributions characterize atmospheric turbulence. The temperature structure parameter, $C_T{}^2$, and the index of refraction structure parameter, $C_n{}^2$, quantify two density distributions that arise in acoustic and electromagnetic propagation. [Ref. 1]

The density distributions quantified by $C_n{}^2$ and $C_T{}^2$ induce phase fluctuations in propagating electromagnetic waves. The density distributions of atmospheric turbulence degrade the spatial and temporal coherences of initially coherent electromagnetic radiation. Similar, more pronounced effects occur in the propagation of acoustic waves. [Ref. 1]

A number of techniques for correcting these problems have been proposed, particularly with respect to electromagnetic propagation. These corrective techniques include the use of modular mirrors [Ref. 2], optical phase conjugation [Ref's. 2 and 3], optical and digital signal processing [Ref. 4], and simple avoidance of turbulent air masses. The appropriate timing of transmissions or suitable location of transmission and reception facilities accomplishes avoidance [Ref's. 5 and 6]. These techniques can be applied alone or in combinations.

A knowledge of the history of the density or refractive structure along a proposed transmission path is required to effectively employ a number of these compensatory techniques. Time-averaged C_T^2 or C_n^2 profiles charactize this history [Ref's. 1 and 7].

Wroblewski and Weingartner developed and Moxcey refined a high resolution, computer controlled acoustic sounder, or echosounder, system. Their computer controlled echosounder system uses a transmitted pulse of acoustic energy as a probe of atmospheric structure. The interaction of the acoustic wave packet with atmospheric turbulence is used to develop a time-averaged C_T^2 profile for a short spatial range [Ref's. 5, 6 and 8]. The acoustic sounder may also be used to verify certain long range optical measurements of turbulence [Ref. 6].

Development of the C_T^2 profile uses a C_T^2 expression developed from the echosonde equation summarized by Neff [Ref. 7] and the empirical acoustic backscatter cross section per unit volume expression of Tatarski [Ref. 9] [Ref's. 5 and 6]. The accuracy of the absolute values of this C_T^2 profile depends on the calibration of the acoustic antenna parameters contained in the C_T^2 expression. The antenna parameters of interest are E_t, the efficiency of conversion of electrical to acoustic power, and E_r, the efficiency of conversion of acoustic power to electrical power.

In this thesis acoustic energy backscattered off acoustically hard spheres was used to determine E_r and E_t. Acoustic range equations were software implemented to perform the necessary calculations. Data and calculational results provided first order verification of the calibration process.

II THEORETICAL BACKGROUND

A. C_T^2 REVIEW

Wroblewski and Weingartner applied the echosonde equation,

$$P_r = E_r\left[P_t E_t\right]\left[e^{-2\alpha R}\right]\left[\frac{c\tau}{2}\,\sigma_0(R,f)\right]\left[\frac{A}{R^2}\,G\right], \qquad (1)$$

summarized by Neff [Ref. 7] and the acoustic backscattering cross section per unit volume expression given by Tatarski [Ref. 9] as

$$\sigma_0(R,f) = 0.0039\, k^{1/3}\, \frac{C_T^2}{T_0^2} \qquad (2)$$

to develop the volume averaged expression

$$C_T^2 = \frac{1}{0.0039}\,\frac{1}{E_r E_t}\,\frac{T_0^2}{k^{1/3}}\,\frac{2}{c\tau}\,\frac{1}{AG}\,\frac{P_r}{P_t}\,R^2\, e^{2\alpha R}, \qquad (3)$$

where

- P_r is the electrical power of the reflected signal returned to the acoustical antenna array,
- E_r is the efficiency of conversion of the returned acoustic power to P_r,
- P_t is the electrical power supplied to the acoustical array,
- E_t is the efficiency of conversion of P_t to acoustic power,
- α is the average attenuation per unit distance along the transmission path,
- R is the range from the array to the target or backscattering volume,

- c is the average speed of sound along the transmission path,
- τ is the length in time of the transmitted acoustic pulse,
- A is the antenna's aperture area,
- G is the antenna's effective aperture factor,
- k is the wavenumber of the incident acoustic energy, and
- T_0 is the average temperature (in degrees Kelvin) along the transmission path.

[Ref's. 5 and 6]

The conditions at the time and location of each application determine the value of the terms T_0, k, c, τ, P_r, P_t, R and α in the C_T^2 expression. The terms A and G depend on the antenna design.

Adverse field effects such as dust, debris and vibration loosened connections cause the remaining two terms, E_r and E_t, to change over the life of the system. This chapter develops the theoretical expressions supporting a determination of E_r and E_t.

B. ACOUSTIC RANGE EQUATIONS AND EFFICIENCIES

In the following two sections the acoustic range equation of Neff [Ref. 7] is re-developed for targets of small area relative to the cross-section of the ensonifying field. Neff [Ref. 7], Skolnik [Ref. 10], and Probert-Jones [Ref. 11] were used for guidance.

1. One Way Range Equation and $E_t G_0$

As defined previously, the electrical power supplied to the acoustic array is P_t. It is converted at an efficiency E_t to an emitted acoustic power,

$$P_a = P_t E_t . \quad (4)$$

P_a is spread over a 4π solid angle.

A far field range R is defined by the condition

$$\lambda/2 \gg |L| - |R| \quad (5)$$

for the geometry in Figure 1. λ is the wavelength of the emitted acoustic wave.

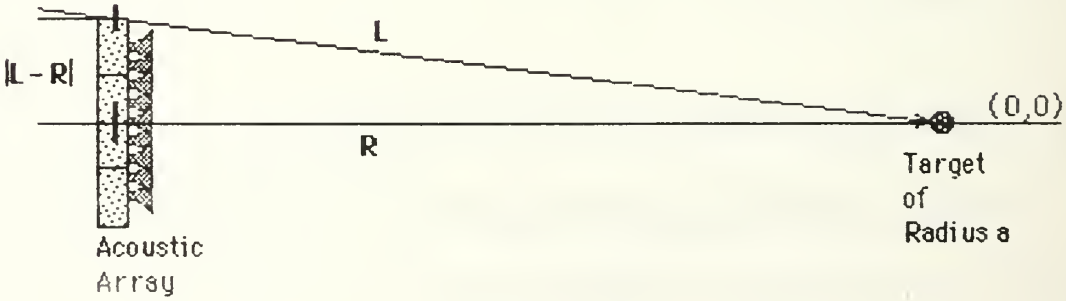


Figure 1: Calibration (Far Field) Geometry

The acoustic intensity,

$$I_a = P_a / 4\pi R^2, \qquad (6)$$

describes the emitted acoustic power density of an isotropic emitter at a far field range R. I_a is in units of [W/m^2].

If the emitted power is directional, the intensity I_a is modulated by a geometrical gain,

$$G(\Omega) = G_0 |F(\Omega)|^2 = G_0 |F(\phi,\theta)|^2. \qquad (7)$$

Note this is not the same G found in equation (1). Now,

$$|F(0,0)|^2 = 1, \qquad (8)$$

and G_0 is thefore the centerline gain of the acoustic array. [Ref. 11] The centerline direction (0,0) is taken to be normal to the face of the acoustic array. The acoustic intensity I_a is now written

$$I_a = \frac{P_a}{4\pi R^2} G(\Omega). \qquad (9)$$

In a real propagating medium intensity attenuation occurs along the transmission path. The radial term $e^{-\alpha R}$ accounts for this attenuation when the atmospheric path is assumed to be reasonably homogeneous. The acoustic intensity is then

$$I_a = \frac{P_a}{4\pi R^2} G(\Omega) e^{-\alpha R} . \qquad (10)$$

A target of cross sectional area A_{tgt} intercepts a power

$$P_{tgt} = I_a A_{tgt} . \qquad (11)$$

Expanding this expression with equation (10) yields

$$P_{tgt} = \left[\frac{P_a}{4\pi}\right]\left[e^{-\alpha R} G(\Omega)\right]\left[\frac{A_{tgt}}{R^2}\right] . \qquad (12)$$

For a target of cross sectional area A_{tgt} with a sufficiently small solid angle A_{tgt}/R^2 little change in intensity occurs about the direction Ω. Equation (12) is now arranged as a one way range equation along the centerline direction (0,0);

$$R = \left[\frac{P_t E_t}{4\pi} e^{-\alpha R} G_0 \frac{A_{tgt}}{P_{tgt}}\right]^{\frac{1}{2}} . \qquad (13)$$

Isolating $E_t G_o$ in the above expression yields

$$E_t G_0 = \frac{4\pi R^2}{P_t} e^{\alpha R} \frac{P_{tgt}}{A_{tgt}} . \qquad (14)$$

Electrical power is expressed as

$$P = \frac{{V_{rms}}^2}{Z} = \frac{V_{ms}}{Z} , \qquad (15)$$

where

- V_{rms} is the empirical root mean square voltage,
- V_{ms} is ${V_{rms}}^2$, and
- Z is the electrical impedance.

$E_t G_0$ is now written

$$E_t G_0 = 4\pi R^2 \frac{Z}{(V_{ms})_t} e^{\alpha R} \frac{P_{tgt}}{A_{tgt}} . \qquad (16)$$

The ratio P_{tgt}/A_{tgt} is defined as the acoustic intensity I_r received by the target. Therefore,

$$E_t G_0 = 4\pi R^2 \frac{Z}{(V_{ms})_t} e^{\alpha R} \quad I_r \quad . \qquad (17)$$

2. Two Way Range Equation and Er

For an acoustically hard target the total power reflected is P_{tgt}.

The power reflected from the target surface has a normalized, directional intensity distribution described by the differential scattering cross section, σ.

The direction from the target to the array is the backscattered direction. The differential scattering cross section in the backscattered direction is called the normalized backscattered cross section, σ_b. σ_b is defined as the power reflected toward the source per unit solid angle, normalized by the incident power density over a 4π solid angle.

The backscattered intensity from the target is

$$I_{tgt} = \frac{P_{tgt}}{4\pi R^2} \sigma_b \quad . \tag{18}$$

An array of aperture area A at range R intercepts an attenuated, received acoustic power,

$$P_{ra} = I_{tgt} A e^{-\alpha R} . \tag{19}$$

Substituting equation (18) into equation (19) yields

$$P_{ra} = \frac{P_{tgt}}{4\pi R^2} \sigma_b \; e^{-\alpha R} \; A \quad . \tag{20}$$

P_{ra} is converted to returned electrical power, P_r, at a return efficiency, E_r;

$$P_r = P_{ra} E_r . \tag{21}$$

Since electrical power is expressed as

$$P = \frac{V_{rms}^{\;2}}{Z} = \frac{V_{ms}}{Z} , \tag{15}$$

electrical power representing the returned acoustic signal from a target placed along an emitter's direction (0,0) is written as

$$\frac{(V_{ms})_r}{Z_r} = \frac{(V_{ms})_t}{Z_t}\left[\frac{E_t G_0}{4\pi}\; e^{-\alpha R}\; \frac{A_{tgt}}{R^2}\right]\left[\frac{E_r \sigma_b}{4\pi}\; e^{-\alpha R}\; \frac{A}{R^2}\right] . \tag{22}$$

For a passive, linear acoustic array circuit the concept of impedance reciprocity is invoked [Ref. 12] to yield a two way range equation for scattering from a small target;

$$R = \left[\frac{(V_{ms})_t}{(V_{ms})_r}\; E_t E_r\; \frac{G_0 \sigma_b}{(4\pi)^2}\; e^{-2\alpha R}\; A\, A_{tgt}\right]^{\frac{1}{4}} . \tag{23}$$

This is written to isolate E_r;

$$E_r = \frac{(V_{ms})_r}{(V_{ms})_t}\; \frac{(4\pi R^2)^2}{E_t G_0}\; \frac{e^{2\alpha R}}{A\, A_{tgt}}\; \frac{1}{\sigma_b} . \tag{24}$$

A part of the return signal's power is unwanted noise. Subtracting noise power from the returned acoustic power yields

$$E_r = \frac{(V_{ms})_r - (V_{ms})_n}{(V_{ms})_t}\; \frac{(4\pi R^2)^2}{E_t G_0}\; \frac{e^{2\alpha R}}{A\, A_{tgt}}\; \frac{1}{\sigma_b} . \tag{25}$$

As return voltages are quite small they are analyzed after electrical amplification. Calling the electrical amplification a gain G_e allows introduction of the final modification to E_r:

$$E_r = \frac{(V_{ms})_r - (V_{ms})_n}{(V_{ms})_t \; G_e^2} \; \frac{(4\pi R^2)^2}{E_t G_0} \; \frac{e^{2\alpha R}}{A \; A_{tgt}} \; \frac{1}{\sigma_b} \; . \qquad (26)$$

C. ATTENUATION AND RANGE DETERMINATION

Employing equations (17) and (26) in the calibration process requires calculating the attenuation α and the range R from the array to the target.

1. Attenuation

The calculated attenuation α for an atmospheric propagation path is assumed to be the combined result of molecular and classical absorption in the atmosphere. The classical absorption α_{cl} is attributed to heat conduction and viscous effects while the larger molecular absorption α_{mol} is attributed to the excitation of internal energy modes of atmospheric gases by the propagating sound energy. [Ref. 7]

Information from Neff [Ref. 7], Businger [Ref. 13], Neiburger [Ref. 14], and Fuller [Ref. 15] is applied to make a reasonable determination of α. The determination is made using the average measured temperature

T_c in degrees Celsius along the propagation path, the measured atmospheric pressure P in millibars, and the measured relative humidity Rh in percent.

The molecular attenuation coefficient, α_{mol}, is expressed by the empirical relationship

$$\alpha_{mol} = \frac{\alpha_{max}}{304.8}\left[(0.18\, f_{ratio})^2 + \left(\frac{2(f_{ratio})^2}{1 + f_{ratio}^2}\right)^2\right]^{\frac{1}{2}} . \qquad (27)$$

α_{mol} is in units of [-dB/m].

Now,

$$\alpha_{max} = 0.0078\, f_m\, (T*)^{-2.5}\, e^{7.77(1-1/T*)} \qquad (28)$$

is the maximum absorption at

$$f_m = \frac{(10 + 6600\,(100\frac{e}{P}) + 44{,}400\,(100\frac{e}{P})^2)\, P*}{(T*)^{0.8}} . \qquad (29)$$

f_m is in units of [Hz].

Also,

$$\frac{e}{P} = \frac{E_s\, Rh}{P - E_s(1 + Rh)} \qquad (30)$$

is the mole ratio of water vapor in the atmosphere. The water vapor pressure e is expressed in millibars.

$$E_s = 10^{(9.4 - 2353/T_0)} \qquad (31)$$

expresses the saturation vapor pressure E_s. T_0 is the average absolute temperature along the transmission path, as defined in Section A.

$$T^* = (1.8\ T_c + 492) / 519\ , \qquad (32)$$

and

$$P^* = P / 1014\ . \qquad (33)$$

Finally,

$$f_{ratio} = f / f_m\ , \qquad (34)$$

where f is the operating frequency of the echosounder in [Hz]. [Ref's. 7, 13, 14 and15]

The classical attenuation coefficient, α_{cl}, is approximated by

$$\alpha_{cl} = 1.74 * 10^{-10} (f)^2\ . \qquad (35)$$

α_{cl} is in units of [-dB/m]. [Ref. 7]

For a total attenuation

$$\alpha_T = \alpha_{cl} + \alpha_{mol} \qquad (36)$$

and the relationship

$$-\alpha_T R = 10 \log e^{-\alpha R} \qquad (37)$$

the average attenuation per meter is finally written as

$$\alpha = \frac{\alpha_{cl} + \alpha_{mol}}{10} \ln 10 \qquad (38)$$

Equation (37) is derived from unit definitions and equation one of Neff [Ref. 7].

2. Range

The coordinate system for the backscattered cross section determination uses the center of the target spheres as the coordinate system origin. Therefore, the radius a of a target sphere and the time of return t_r of an acoustic signal from the acoustic echosounder are used to determine the range R from the target to the echosounder;

$$R = \frac{ct_r}{2} + a \ . \qquad (39)$$

c is the average speed of sound along the transmission path, and

$$t_r = t_{receive} - t_{transmit} \ . \qquad (40)$$

For the range of temperatures expected in normal echosounder operation the speed of sound in meter per seconds in dry air is

$$c_{dry} = 20.05 \sqrt{T_0} \ . \qquad (41)$$

For moist air c becomes

$$c = c_{moist} = c_{dry}(1 + 0.14\,e/P) , \qquad (42)$$

where e/P is defined in equation (30). [Ref. 1]

D. BACKSCATTERED CROSS SECTION DETERMINATION

Employing equations (17) and (26) in the calibration process also requires calculating σ_b, the backscattered cross section. Recall from Paragraph B.2 that the backscattered cross section is the differential scattering cross section in the direction of the energy source.

1. Initial Conditions and Assumptions

In determining the differential scattering cross section σ it is assumed the incident wave state is unchanged after scattering. That is, for a stationary, acoustically hard target it is assumed

$$k = k_{incident} = k_{scattered} , \qquad (43)$$

and that incident k has good definition, i.e. the incident wave can be considered monochromatic [Ref. 16].

Selection of acoustically hard spheres as targets further simplifies the determination of the differential scattering cross section solution. This target selection sets the boundary condition at the target surface as the Neumann boundary condition

$$\frac{\partial V}{\partial n} = 0 . \tag{44}$$

V is the velocity potential defined by the acoustic propagation velocity c as

$$c = \nabla V . \tag{45}$$

n lies along the direction normal to the target surface. Also,

$$p = -\rho_0 \frac{\partial V}{\partial t} \tag{46}$$

for the pressure p and rest density ρ_0 of the propagating medium. [Ref. 16].

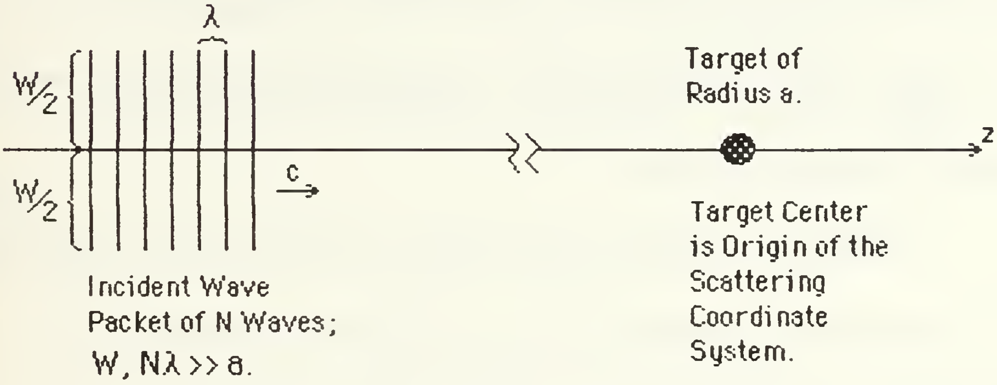

λ
W/2
W/2
c
Target of Radius a.
z
Target Center is Origin of the Scattering Coordinate System.
Incident Wave Packet of N Waves; W, Nλ >> a.

Figure 2: Incident Wave and Target Description

The target sphere is aligned along the centroid of the main acoustic lobe of the acoustic arrray echosounder, that is, along the direction (0,0), normal to the face of the array. The target sphere is selected to be sufficiently small with respect to incident beam dimensions so that an essentially uniform intensity is intercepted by the sphere [Ref. 17]. Since the target intercepts a very small proportion of the centroid of the transmitted sounding lobe no accommodation for sounding lobe geometry over the target's angular area is made. Acoustic gain is simply taken as G_0.

Target placement is also at a far field range. This allows spherical incident waves to be treated as planar incident waves and places the target outside the range associated with the ring time of the echosounder [Ref. 8]. Placement of the target is also close enough to the array to support an assumption of transmission medium homogeneity along the transmission path.

Finally, the potential describing the target falls off faster than 1/r. The describing potential for an acoustically hard sphere with radius a is

$$V(\mathbf{r}) = \begin{cases} \infty & r \leq a \\ 0 & r \geq a \end{cases} \quad . \tag{47}$$

Thus, for $r > a$ $V(\mathbf{r})$ falls off faster than $1/r$.

Note that with the exception of the specific surface boundary condition for acoustic waves the above conditions and assumptions closely correspond to those for plane electromagnetic waves incident on a perfectly conducting sphere, and for perfectly elastic nuclear scattering in a center of mass coordinate system. [Ref's. 16, 18, 19 and 20]

2. The Incident Wave

The asymptotic form of an incident plane wave approaching a scattering target but beyond the influence of the target's potential V(**r**) is the undisturbed, or free, plane wave. In the spherical polar coordinates (r,ϕ,θ) illustrated in Figure 3 a free, monochromatic plane wave of unit amplitude propagating in the direction Ω_0 given by (ϕ_0,θ_0) is

$$\psi(\mathbf{r},t) = e^{ikr[\cos(\phi_0)\cos(\phi) + \sin(\phi_0)\cos(\theta - \theta_0)]}\, e^{-i\omega t} . \quad (48)$$

[Ref. 16]

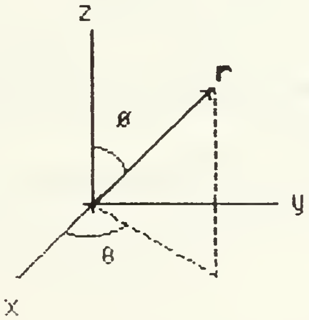


Figure 3: Spherical Polar Coordinates

For a wave of unit amplitude incident along the z axis , $\Omega_0 = 0$ and the free wave $f(\mathbf{r},t)$ is presented as

$$f(\mathbf{r},t) = e^{ikr\cos(\phi)}\, e^{-i\omega t} = e^{ikz}\, e^{-i\omega t} . \qquad (49)$$

Note this is a solution with axial symmetry for the free wave equation [Ref. 18].

The wave equation describing the behavior of the undisturbed or free monochromatic wave $f(\mathbf{r},t)$ is

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \right) f(\mathbf{r},t) = 0 \, . \qquad (50)$$

c is the speed of propagation of the free wave. If the wave function $f(\mathbf{r},t)$ has a simple harmonic time dependence, i.e.,

$$f(\mathbf{r},t) = e^{-i\omega t} f(\mathbf{r}) , \qquad (51)$$

then for a wave number

$$k \equiv \omega/c = 2\pi/\lambda \qquad (52)$$

the free wave equation becomes the Helmholtz equation

$$[\nabla^2 + k^2] f(\mathbf{r},t) = 0 \qquad (53)$$

[Ref. 21].

The stationary solution $f(\mathbf{r})$ of the free wave equation can be written as the product

$$f(\mathbf{r}) = f_1(q_1)\, f_2(q_2)\, f_3(q_3) \qquad (54)$$

for orthogonal curvilinear coordinates q_1, q_2, q_3 [Ref. 16]. Recall an undisturbed plane wave has a cylindrical symmetry about the axis of propagation [Ref. 22]. Using the spherical polar coordinates

$$q_1 = r, \quad q_2 = \phi, \quad q_3 = \theta, \qquad (55)$$

the stationary solution describing an undisturbed plane wave propagating along the z axis is

$$f(\mathbf{r}) = f_1(r)\, f_2(\phi) \; . \qquad (56)$$

Substituting $f(\mathbf{r})$ into the free wave equation expressed in spherical coordinates and separating the coordinate dependencies yields the separated free wave equation,

$$(kr)^2 + \frac{1}{f_1}\frac{d}{dr} r^2 \frac{d}{dr} f_1 = -\frac{1}{f_2 \sin\phi}\frac{d}{d\phi}\left(\sin\phi \frac{d}{d\phi} f_2\right). \qquad (57)$$

Using a separation constant of $l(l+1)$ the equation for the angular dependency of the free wave becomes

$$\frac{1}{f_2 \sin\phi}\frac{d}{d\phi}\left(\sin\phi \frac{d}{d\phi} f_2\right) + l(l + 1) = 0 \; . \qquad (58)$$

Letting

$$\cos(\phi) = x \qquad (59)$$

in the angular equation yields

$$\left[(1 - x^2)\frac{d^2}{dx^2} - 2x\frac{d}{dx} + l(l + 1)\right] f_2 = 0\,. \qquad (60)$$

The previous equation is known as the Legendre differential equation [Ref. 21]. The most useful solutions to the Legendre differential equation are the Legendre polynomials, designated by $P_l(x)$. They can be generated by the recursion relationships

$$P_0(x) = 1, \qquad (61a)$$

$$P_1(x) = x \qquad (61b)$$

and

$$P_l(x) = [2 - 1/l]\, x\, P_{l-1}(x) - [1 - 1/l]\, P_{l-2}(x) \qquad (61c)$$

[Ref. 21]. Therefore,

$$f_{2,l}(\phi) = P_l(\cos\phi)\,. \qquad (62)$$

From equation (57), the equation describing the radial dependency of the free wave is

$$\left\{\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} + \left[k^2 - \frac{l(l+1)}{r^2}\right]\right\} f_1 = 0\,. \qquad (63)$$

Changing scale to the variable s = kr yields the differential equation

$$\left\{ \frac{d^2}{ds^2} + \frac{2}{s}\frac{d}{ds} + \left[1 - \frac{l(l+1)}{s^2} \right] \right\} f_l = 0 \qquad (64)$$

[Ref. 17].

This differential equation has the spherical Bessel function $j_l(s)$ with amplitude A_l as a solution;

$$f_{l,l}(s) = A_l \, j_l(s) \qquad (65)$$

[Ref. 18].

The stationary solution of the free wave equation in spherical coordinates is now written as the partial wave

$$f_l(r) = P_l(\cos \varnothing) \, [A_l \, j_l(kr)] \, . \qquad (66)$$

The most general form of this free wave equation solution is a Fourier-Bessel series,

$$f(r,\varnothing) = \sum_{l=0}^{\infty} A_l \, j_l(kr) \, P_l(\cos \varnothing) \qquad (67)$$

[Ref's. 17 and 21].

A plane, free wave of unit amplitude propagating in the direction $\Omega_0 = 0$ is then

$$e^{ikz} = \sum_{l=0}^{\infty} A_l \, j_l(kr) \, P_l(\cos \varnothing) \ . \qquad (68)$$

The norm of the Legendre polynomials, the asymptotic expression

$$j_l(kr) \sim \frac{\sin(kr - \frac{\pi}{2} l)}{kr} \qquad (69)$$

and the process detailed in Elton [Ref. 18] are used to determine A_l;

$$A_l = i^l\,[2l+1]\ . \qquad (70)$$

Therefore,

$$e^{ikz} = \sum_{l=0}^{\infty} i^l\,[2l+1]\, j_l(kr)\, P_l(\cos \theta) \qquad (71)$$

[Ref's. 18 and 21].

3. Detection Conditions and Assumptions

As the dimensions of the incident acoustic wave packet are greater than the size of the target it is expected that a portion of the incident wave packet continues to propagate past the target. The total or disturbed wave description is then a superposition of the incident and scattered wave descriptions. [Ref. 17]

The acoustic array serves as both transmitter and receiver, or detector. This keeps the detector outside the path of continued propagation of the incident wave packet. [Ref. 17] Therefore, only the scattered wave description is of interest in the calibration of the array.

The placement of the target at a far field range simultaneously places the detector in the region of asymptotic behavior of the scattered wave.

4. The Disturbed Wave

Once an incident wave described by the velocity potential $\mathcal{V}(\mathbf{r})$ enters the region of influence of a scattering target's spherical potential $V(\mathbf{r})$ the wave equation becomes the disturbed wave equation

$$\left\{ \nabla^2 + [k^2 - V(r)] \right\} \mathcal{V}(\mathbf{r}) = 0 \qquad (72)$$

[Ref. 17]. Comparison of the form of the disturbed wave equation to the free wave equation gives an expectation of a solution of the form

$$\mathcal{V}(\mathbf{r}) = \mathcal{V}_1(r)\, \mathcal{V}_2(\phi) \ . \qquad (73)$$

Substituting $\mathcal{V}(\mathbf{r})$ into the disturbed wave equation and separating variables yields

$$-\frac{1}{\mathcal{V}_2 \sin\phi}\frac{d}{d\phi}\left(\sin\phi\frac{d}{d\phi}\right)\mathcal{V}_2 = r^2[k^2 - V(r)] + \frac{1}{\mathcal{V}_1}\frac{d}{dr} r^2 \frac{d}{dr}\mathcal{V}_1 \ . \qquad (74)$$

The angular equation for the disturbed wave is identical in form to the angular equation for the free wave and independent of the Neumann boundary condition $\partial \mathcal{V}/\partial r = 0$. Therefore,

$$\mathcal{V}_{2,l}(\phi) = P_l(\cos\phi) \ . \qquad (75)$$

Using the previous substitution of s = kr yields the now modified Bessel equation as the radial equation for the disturbed wave;

$$\left\{ \frac{d^2}{ds^2} + \frac{2}{s}\frac{d}{ds} + \left[1 - \frac{l(l+1) + V(r)}{s^2} \right]\right\} \mathcal{V}_l = 0 \ . \qquad (76)$$

Following the solution development in Elton [Ref. 18] and applying the Neumann boundary condition at r = a leads to a solution

$$\mathcal{V}_{l,l}(r) = i^l \, [2l + 1] \, [j_l(kr) - a_l' \, h_l^{(1)}(kr)] \ . \qquad (77)$$

The relationship

$$a_l' = i \, e^{-i\delta_l} \sin \delta_l = \frac{j_l'(ka)}{h_l^{(1)'}(ka)} \qquad (78)$$

is defined from Morse [Ref. 17] and Bowman [Ref. 16]. Additionally,

- the $j_l(kr)$ are the spherical Bessel functions of the first kind, argument kr, and order l,
- $h_l^{(1)}(kr)$ are the Hankel functions of argument kr and order l,
- the δ_l are the phase shifts of the usual partial wave description for spherical scattering solutions [Ref's. 17 and 20],
- the $j_l'(ka)$ are the first derivatives of the $j_l(kr)$, taken with respect to the argument kr and evaluated at r = a,
- $h_l^{(1)'}(ka) = j_l'(ka) + i \, y_l'(ka)$, and (79)
- the $y_l'(ka)$ are the first derivatives of the $y_l(kr)$, the spherical Bessel functions of the second kind with argument kr and order l, taken with respect to the argument kr and evaluated at r = a.

The general series solution for the total disturbed wave $\Psi(\mathbf{r})$ is then

$$\Psi(\mathbf{r}) = \sum_{l=0}^{\infty} i^{l}\,[2l + 1]\,P_{l}(\cos\phi)\,[j_{l}(kr) - a_{l}'\,h_{l}^{(1)}(kr)]\ . \quad (80)$$

5. The Scattered Wave and Its Amplitude

The general asymptotic form for a disturbed wave $\Psi(r,\Omega)$ resulting from a plane wave $\Psi_{i}(r,\Omega)$ incident along $\Omega_{0} = 0$ and scattered by a spherical potential V(r) is

$$\Psi(r,\Omega) \cong e^{ikz} + s(\Omega)\,e^{ikr}/r \qquad (81)$$

[Ref. 18]. This description of the asymptotic disturbed wave is a superposition of the incident plane wave e^{ikz} and a spherical, scattered wave $s(\Omega)e^{ikr}/r$. The term $s(\Omega)$ is called the scattering amplitude [Ref's. 18, 19 and 22].

Now, recalling the series equivalence for the plane wave incident along Ω_{0} in equation (71), and comparing the general series solution for the disturbed wave in equation (80) to its asymptotic form in equation (81) gives

$$s(\Omega)e^{ikr}/r = -\sum_{l=0}^{\infty} i^{l}\,[2l + 1]\,P_{l}(\cos\phi)\,a_{l}'\,h_{l}^{(1)}(kr)\ . \quad (82)$$

This is a series expression for the scattered wave. Therefore,

$$s(\Omega) = \frac{\sqrt{4\pi}}{k}\, i \sum_{l=0}^{\infty} [2l + 1]\, P_l(\cos \phi)\, a_l' \tag{83}$$

[Ref's. 16 and 18].

6. The Backscattered Cross Section

The scattering amplitude $s(\Omega)$ is related to the differential scattering cross section $\sigma(\Omega) = \sigma A_{tgt}$ by

$$\sigma(\Omega) = |s(\Omega)|^2 \tag{84}$$

[Ref's. 18, 19 and 22].

Applying the series expression for the scattering amplitude in equation (83) to the previous equation yields

$$\sigma A_{tgt} = \sigma(\Omega) = \frac{4\pi}{k^2} \left| \sum_{l=0}^{\infty} i\, [2l + 1]\, a_l'\, P_l(\cos \phi) \right|^2 . \tag{85}$$

For the backscattered direction $\phi = \pi$ this gives a backscattered cross section of

$$\sigma_b A_{tgt} = \sigma_B = \frac{4\pi}{k^2} \left| \sum_{l=0}^{\infty} [2l + 1]\, a_l' \right|^2 \tag{86a}$$

[Ref's. 16 and 17]. For the spherical target $A_{tgt} = \pi a^2$, so

$$\sigma_b = \frac{4}{(ka)^2} \left| \sum_{l=0}^{\infty} [2l + 1]\, a_l' \right|^2 . \tag{86b}$$

It should be noted a plane wave of unit amplitude incident from the positive z direction and backscattered along ø = 0 would introduce a factor of $(-1)^l$ into the previous sum [Ref. 16].

E. SPHERICAL BESSEL FUNCTIONS AND THEIR DERIVATIVES

Implementation of the previous equation necessitates calculation of the a_l'. The recursion relations

$$j_0'(ka) = [-j_0(ka) + \cos(ka)]/ka\,, \tag{87a}$$

$$j_l'(ka) = \left\{-[l + 1][j_l(ka)]/ka\right\} + j_{l-1}(ka)\,, \tag{87b}$$

$$y_0'(ka) = [-y_0(ka) + \sin(ka)]/ka \tag{87c}$$

and

$$y_l'(ka) = \left\{-[l + 1][y_l(ka)]/ka\right\} + y_{l-1}(ka) \tag{87d}$$

determine the $j_l'(ka)$ and $y_l'(ka)$ needed for the calculation of a_l' in equation (78) [Ref's. 21 and 23].

Determination of the regular and irregular spherical Bessel functions at r = a supports the use of the recursion relations in equations (87). As ka is real the irregular spherical Bessel functions are stable enough to use the recursion relations

$$y_0(ka) = -[\cos(ka)]/ka\ , \tag{88a}$$

$$y_1(ka) = [y_0(ka) - \sin(ka)]/ka \tag{88b}$$

and

$$y_l(ka) = \left\{[2l - 1][y_{l-1}(ka)]/ka\right\} - y_{l-2}(ka) \tag{88c}$$

in their determination [Ref's. 23 and 24].

Because the size of the argument of the regular spherical Bessel functions may be very small the method of negative continued fractions is appropriate to the fast and accurate determination of a converging solution to these functions [Ref.24]. The regular spherical Bessel function reciprocals are therefore determined by the negative continued fraction calculation

$$\frac{1}{j_l(ka)} = \frac{1}{j_0(ka)}\ \frac{[-][b_{l,1}]\,[b_{l,2},b_{l,1}]\ldots\,[b_{l,1}]\,[b_{l,2},b_{l,1}]\ldots}{[b_{l,2}]\,\ldots\ldots\ldots\,[b_{l,2}]\,[b_{l,3},b_{l,2}]\ldots}\ . \tag{89}$$

Now,

$$b_{l,m} = 2(l + m - 1)/ka \tag{90}$$

and

$$[-][b_{1,m},\ldots,b_{1,2},b_{1,1}] = b_{1,m} - \frac{1}{b_{1,m-1}} - \cdots - \frac{1}{b_{1,2}} - \frac{1}{b_{1,1}}$$

$$= b_{1,m} - \cfrac{1}{b_{1,m-1} - \cfrac{1}{b_{1,m-2} - \cfrac{1}{b_{1,m-3} - \cdots \cfrac{1}{b_{1,2} - \cfrac{1}{b_{1,1}}}}}} \qquad (91)$$

[Ref. 25].

While the regular spherical Bessel function reciprocal calculation of equation (89) would appear to involve an infinite number of terms, the calculation can in fact be terminated when the mth numerator term equals the mth denominator term to the desired precision for the spherical Bessel function's solution. [Ref's. 24 and 25]

III. CALIBRATION PROCESS AND SOFTWARE

Small quantities of data were collected in the anechoic chamber at the Naval Postgraduate School. This data aided first order verification of the calibration process and its supporting software. A brief description of the anechoic chamber is provided in Appendix A.

The calibration process and data development occured in two stages. The first stage used equation (26) and two way propagation path measurements to determine $\sigma_b E_r E_t G_0$. The second stage determined $E_t G_0$ using equation (17) and one way propagation path measurements.

E_r was then determined from $\sigma_b E_r E_t G_0$, $E_t G_0$ and the calculated σ_b of equation (86b).

Because reciprocity was assumed for E_r and E_t

$$E_r E_t = E_r^2 \qquad (92)$$

provided the desired calibration information.

Supporting software was written to perform calculations required for calibration. The next section describes the development of this software.

A. CALIBRATION SOFTWARE DEVELOPMENT

Calibration software was written in BASIC. It was used with a BASIC 4.0 compiler on a Hewlett Packard 310 computer. Computer performance was enhanced by an Infotek floating point processor.

Notation in the programs sometimes differs from that in the thesis text as a result of H.P. keyboard limitations. Variable definitions are located at the beginning of subprograms or modules. These definitions are provided throughout a program as an aid in determining program to text variable equivalence.

The main effort of software development was calculation of backscattered cross sections. This was done with equation (86b). Information available for summation loop verification was differential scattering cross section information. Differential scattering section software was developed first.

1. Differential Scattering Cross Section Software Development

The differential scattering cross sections σ were calculated using equation (85) and equation (78).

To support equation (78) employment the subprograms calculating the values of the Legendre polynomials and the values of the regular and irregular spherical Bessel functions were developed first. Each subprogram was developed and tested separately in test programs. Each test program was designed to provide a tabular output for verification against values found in Abramowitz and Stegun [Ref. 23].

The Legendre values were calculated using equations (61). The irregular spherical Bessel functions were calculated using equations (88). The regular spherical Bessel functions were calculated using equations (89), (90) and (91).

Following verification these subprograms were assembled with subprograms calculating the derivatives of the regular and irregular spherical Bessel functions, the normalized differential scattering cross section σ, the scattering modulus of Bowman [Ref. 16], and with three graphics subprograms. This assembly created the differential scattering cross section program found in Appendix B.

The values of the regular and irregular spherical Bessel function derivatives at $r=a$ were calculated using equations (87).

Differential scattering cross sections were calculated using equation (85). The number of orders l required for the summation calculation in equation (85) was determined by generating tabular outputs and determining the number of orders required for numerical stability to occur to 12 decimal places. This stability process was based on the concept of the phase shift δ_l of each outgoing partial scattered wave becoming progressively smaller with increasing l until a_l' (equation (78)) became negligible for the precision desired [Ref's. 18, 19, 20 and 22].

Tabular outputs covered a ka value range of one to thirty at varying scattering angles, including $\phi = \pi$. This ka value range selection was based on the expected maximum target size to be encountered during actual echosounder employment.

Twenty-one orders greater than the integer value of the argument ka were determined to be required for equation (85)'s summation. The number of orders required was greater than the usually cited relationship

$$ka \approx \sqrt{l(l+1)} \qquad (93)$$

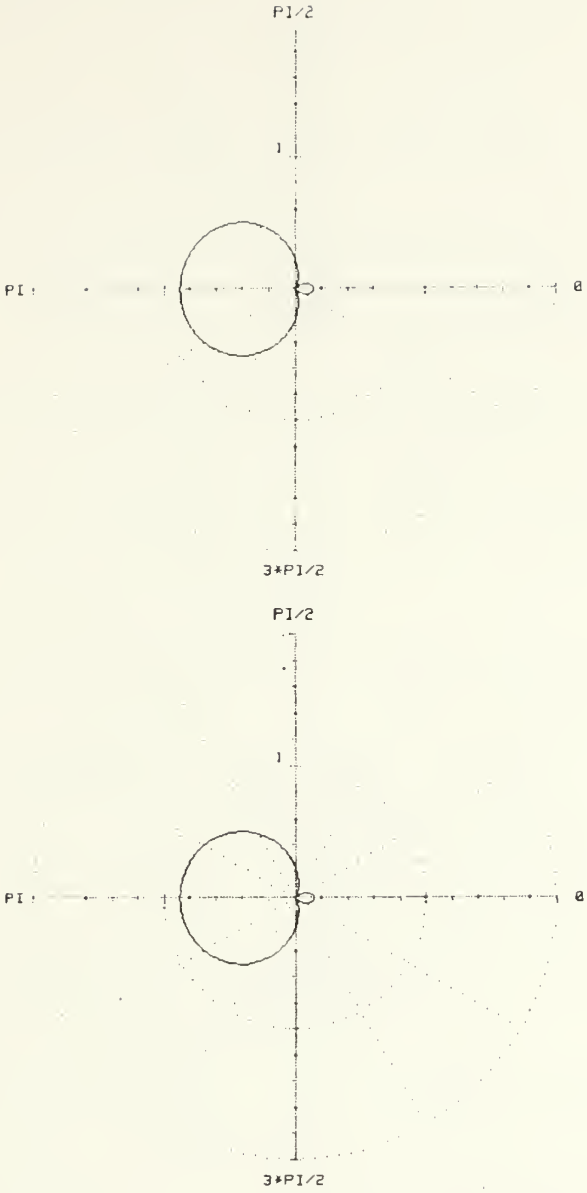
gives. It should be noted that the precision criteria of σ was greater than that normally encountered. [Ref's. 18, 19, 20 and 22]

The graphics subprograms of the differential scattering cross section program provided outputs for comparisons to Figure 80 in

Morse [Ref. 17] and Figure 10.10 of Bowman [Ref. 16]. These comparisons were used to verify the functioning and output of the differential scattering cross section program.

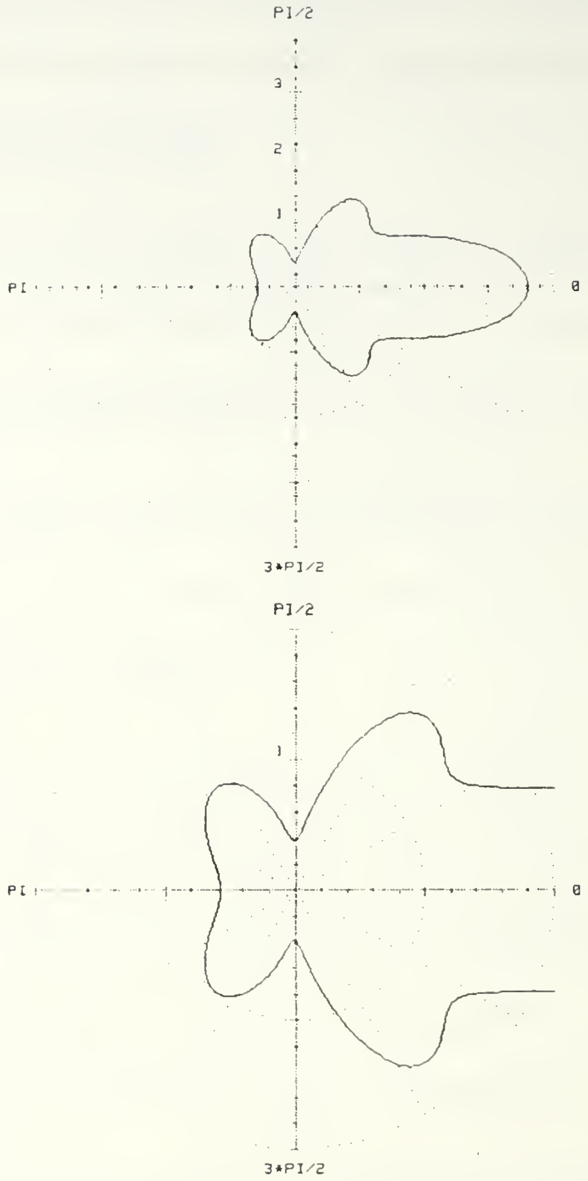
The graphics subprograms used to generate Figure 4 provide linear polar plots of the differential scattering cross section $\sigma(\Omega)$ normalized by the cross sectional area A_{tgt} of the target sphere. These linear polar plots show the normalized differential scattering cross section of acoustically hard spheres ensonified by plane acoustic waves incident from the left, i.e. from $\phi = \pi$.

The first view of each sheet in Figure 4 shows the entire normalized differential scattering cross section pattern, including the forward lobe, or "shadow zone", along $\phi = 0$. The second view of each sheet is a magnified plot showing the scattering detail at a scale comparable to the normalized backscattered cross section value. Note the two views are identical on Sheet 1. The need for magnification with increasing ka becomes apparent on subsequent sheets.



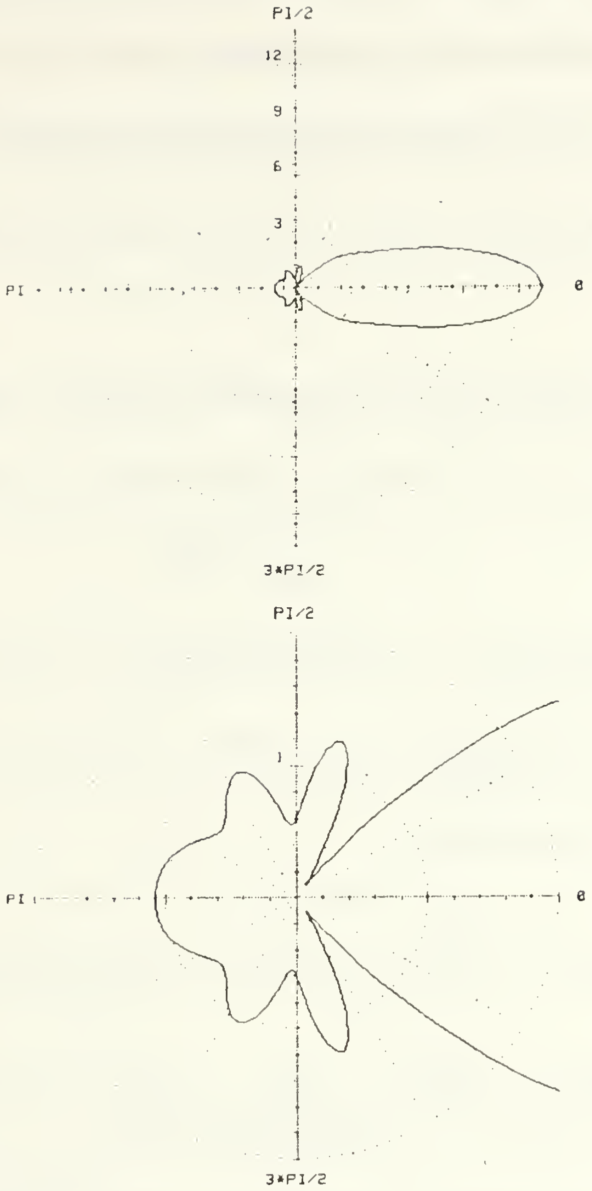
```
C = 343 M/S.
F = 5000 HZ.
A = .0109180290961 M.
K•A = 1 .
NORMALIZED BCKSCTR =  .880073464081
```

Figure 4, Sheet 1 of 3: Differential Scattering Cross Section Program Output for Comparison to Figure 80 of Morse [Ref. 17].



```
C = 343 M/S.
F = 5000 HZ.
A = .0327540872883 M.
K*A = 3 .
NORMALIZED BCKSCTR =   .574927362173
```

Figure 4, Sheet 2 of 3: Differential Scattering Cross Section Program Output for Comparison to Figure 80 of Morse [Ref. 17].



```
C = 343 M/S.
F = 5000 HZ.
A = .0545901454805 M.
K*A = 5 .
NORMALIZED BCKSCTR =  1.08120386888
```

Figure 4, Sheet 3 of 3: Differential Scattering Cross Section Program Output for Comparison to Figure 80 of Morse [Ref. 17].

Figure 4 was used for comparison to Figure 80 of Morse [Ref. 17]. The radial scale of Morse was not graduated so verification consisted of a check for the correct number and approximate angular location of minimums and maximums in the differential scattering cross section patterns.

The graphics subprograms used to generate Figure 5 provide semi-logarithmic plots of the scattering modulus $\mathcal{S}$ of Bowman [Ref. 16].

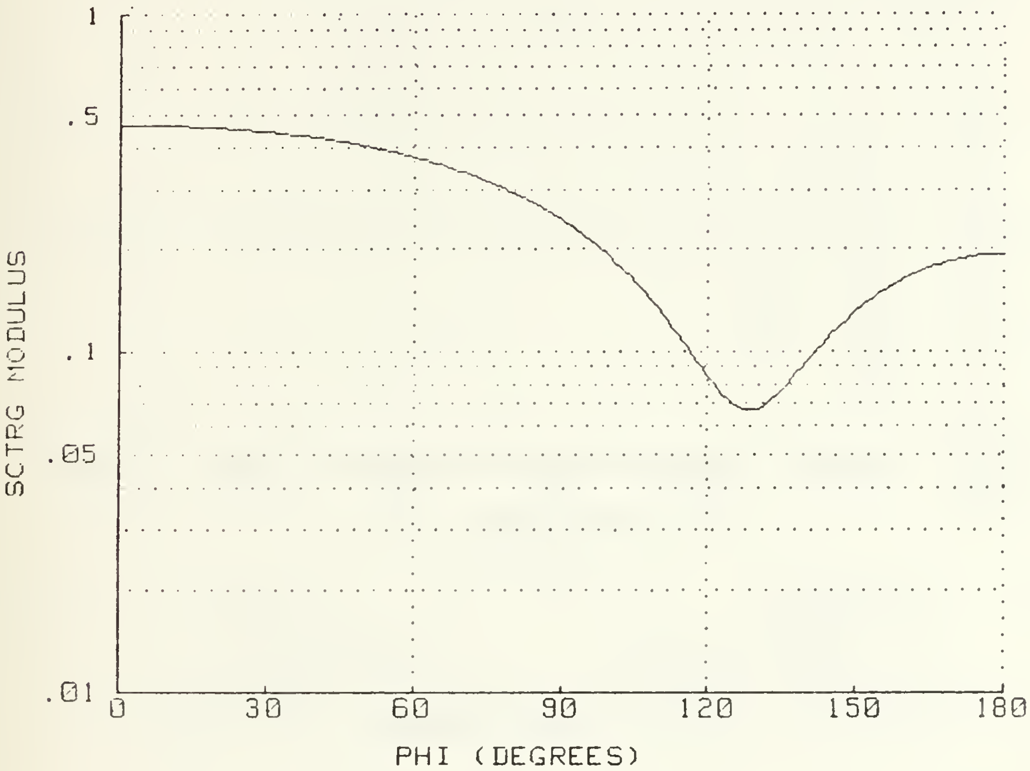
$$\mathcal{S} = k\, s(ø) \,/\, \sqrt{4\pi} \qquad (94)$$

is the scattering modulus $\mathcal{S}$ of Bowman [Ref. 16]. $s(ø)$ is the scattering amplitude $s(\Omega)$, as defined by equation (83), with a factor of $(-1)^l$ introduced into the sum, that is,

$$s(ø) = \frac{\sqrt{4\pi}}{k}\, i \sum_{l=0}^{\infty} (-1)^l\,(2l+1)\,P_l(\cos ø)\,a_l' \,. \qquad (95)$$

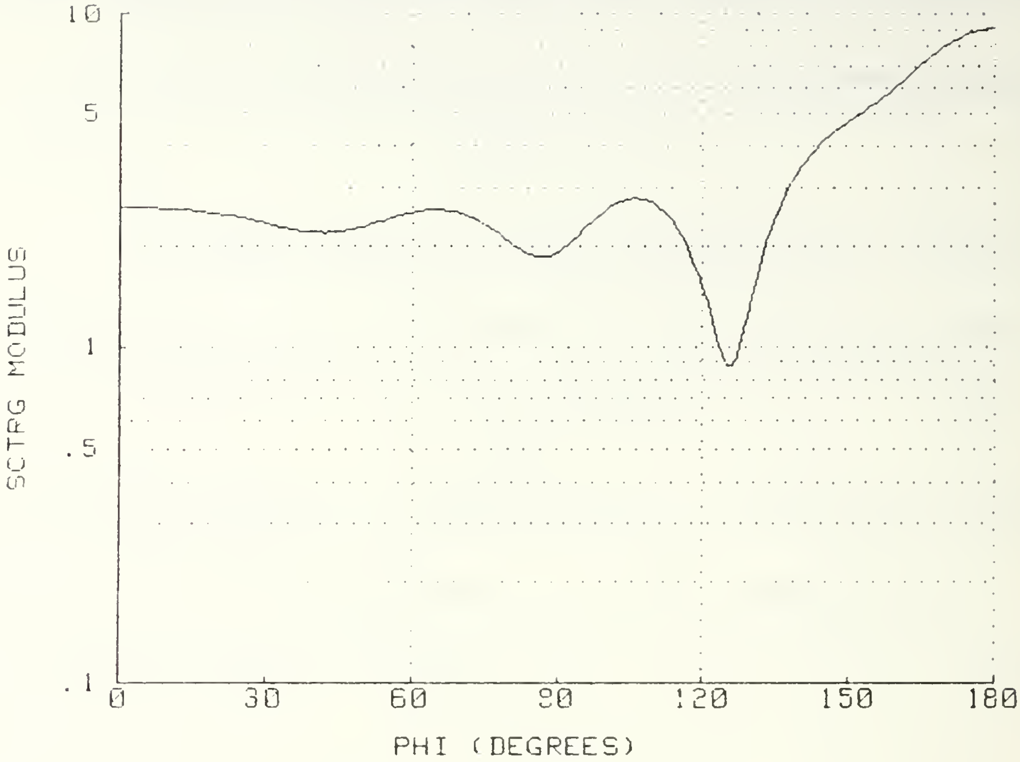
$s(\Omega)$ is normally used to calculate $\sigma(\Omega)$, per equation (84). The scattering modulus $\mathcal{S}$ of Bowman [Ref. 16] could also be used to calculate $\sigma(\Omega)$. Note the introduction of $(-1)^l$ into the summation corresponds to the case of an acoustically hard sphere ensonified by a plane acoustic wave incident from $ø = 0$.

Comparison of Figure 5 to Figure 10.10 of Bowman [Ref. 16] shows a match in both ϕ and S. This graphical match verifies the functioning and output of the differential scattering cross section program of Appendix B.



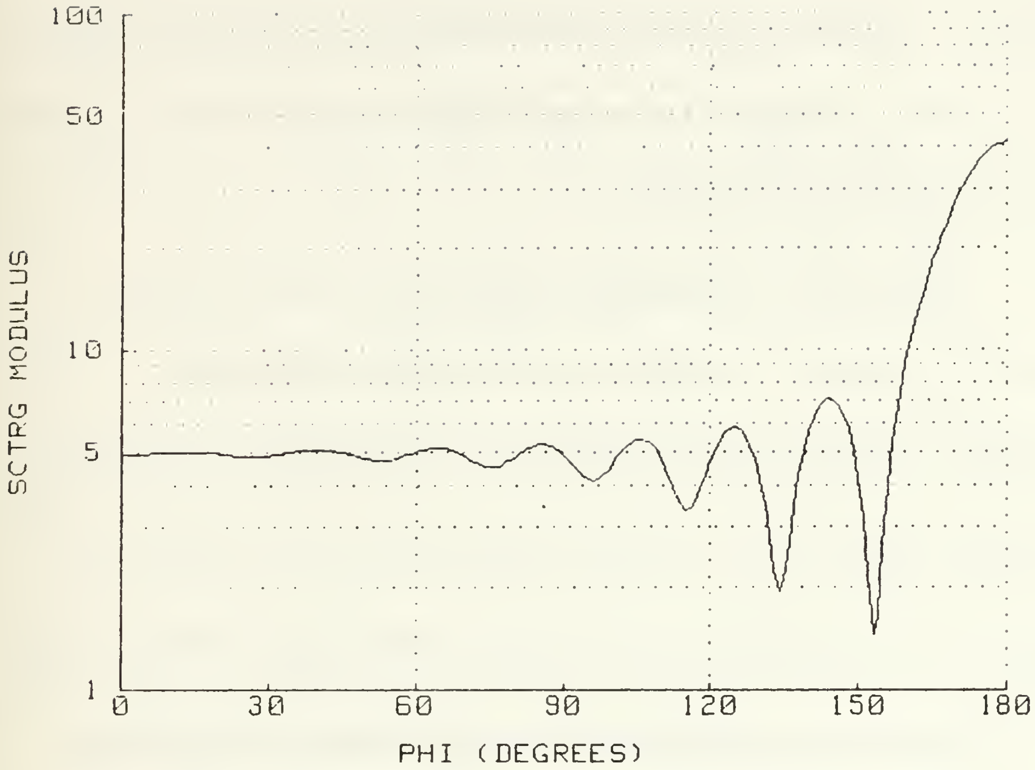
```
C = 343 M/S.
F = 5000 HZ.
A = .0109180290961 M.
K*A = 1 .
NORMALIZED BCKSCTR =   .880073464081
```

Figure 5, Sheet 1 of 3: Differential Scattering Cross Section Program Output for Comparison to Figure 10.10 of Bowman [Ref. 16].



```
C = 343 M/S.
F = 5000 HZ.
A = .0545901454805 M.
K*A = 5 .
NORMALIZED BCKSCTR =  1.08120386888
```

Figure 5, Sheet 2 of 3: Differential Scattering Cross Section Program Output for Comparison to Figure 10.10 of Bowman [Ref. 16].



```
C = 343 M/S.
F = 5000 HZ.
A = .109180290961 M.
K*A = 10 .
NORMALIZED BCKSCTR =  .957615718739
```

Figure 5, Sheet 3 of 3: Differential Scattering Cross Section Program Output for Comparison to Figure 10.10 of Bowman [Ref. 16].

2. Backscattered Cross Section Software Development

The differential scattering cross section program of Appendix B was modified to yield the backscattered cross section program of Appendix C. With the modification the normalized differential scattering cross section is calculated for the backscattered direction only. Equation (86b) is used for this calculation.

Additionally, the graphics subprograms of the differential scattering cross section program were replaced by one graphics subprogram. The new graphics subprogram was designed to provide a rectangular graph of the normalized backscattered cross section, σ_b. σ_b is plotted as a function of ka for a specifiable range of values of ka.

Figure 6 was generated by the backscattered cross section program. It was used for comparison to Figure 10.11 of Bowman [Ref. 16]. This comparison served as a verification of the calculated, normalized backscattered cross section values. Note the ka axis is labeled K*A.

Figure 7 was generated to verify the normalized backscattered cross section profile was approaching a value of one as ka left the Mie scattering regime and entered the "geometric optics" region. Note the ka axis is labeled K*A.

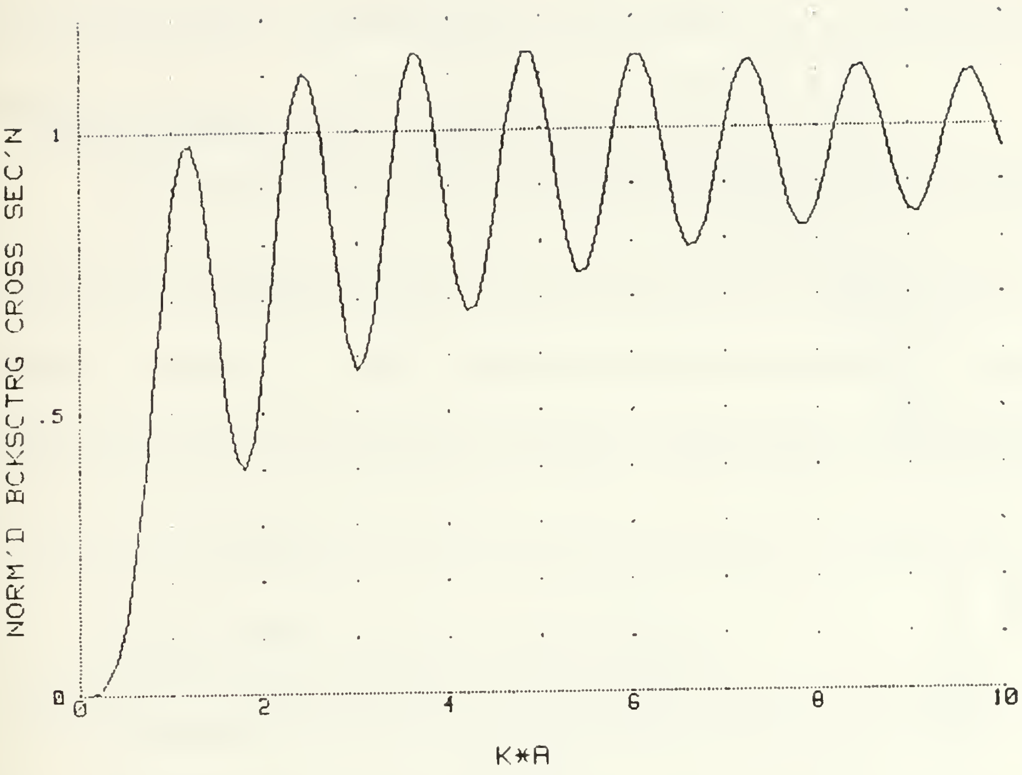


Figure 6: Backscattered Cross Section Program Output for Verification of Calculated Normalized Backscattered Cross Section Values by Comparison to Figure 10.11 of Bowman [Ref. 16].

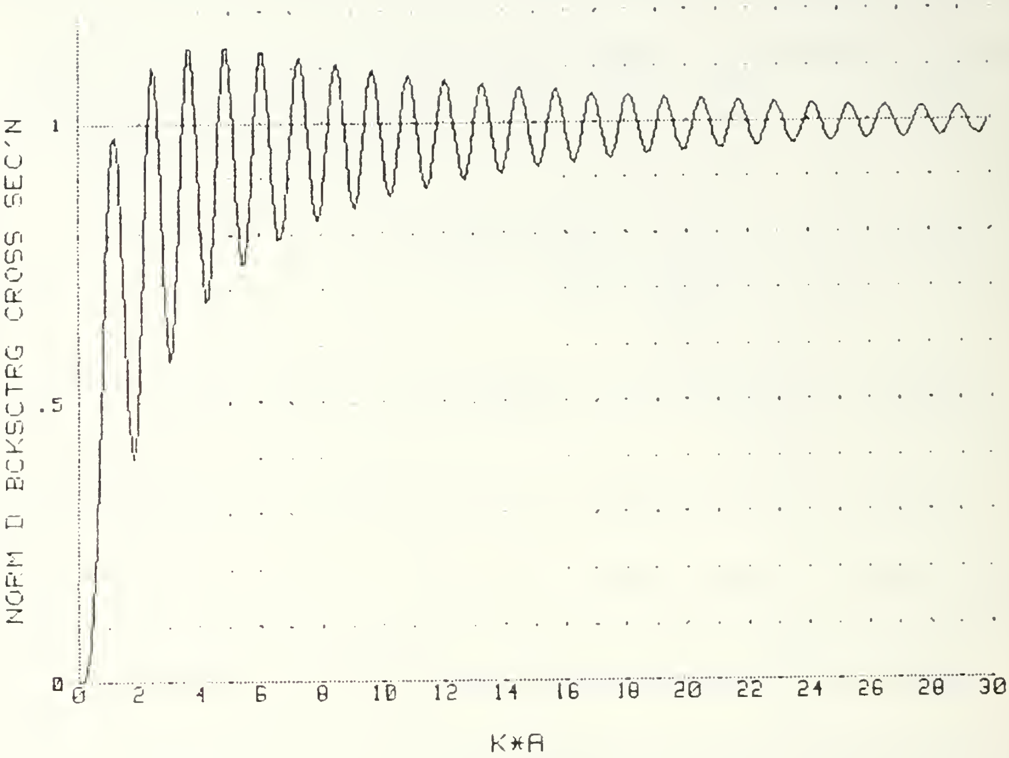


Figure 7: Backscattered Cross Section Program Output for Verification of the Normalized Backscattered Cross Section Asymptotic Behavior with Increasing ka.

3. Acoustic Array Efficiency Software Development

The acoustic array efficiency program in Appendix D determined $E_r E_t$. In general, each major section of Chapter II of this thesis and each set of analyzed data is represented by a subprogram of the acoustic array efficiency program.

The acoustic array efficiency program of Appendix D includes the non-graphics subprograms present in the backscattered cross section program of Appendix C. It also includes subprograms designed to furnish stored data, calculate attenuation and range using equations (27) through (42), and use developed information to calculate $E_t G_0$ with equation (17).

Equation (86b) was used to calculate σ_b.

The appropriate subprogram and main program calculational results were then used in equation (26) to determine E_r. E_r was used in equation (92) to determine $E_r E_t$.

B. DATA ACQUISITION AND SUPPORTING EQUIPMENT

Data collection occured 28 November 1987, 30 November 1987, and 7 December 1987.

Data collected 28 November and 30 November provided information needed for calculation of $\sigma_b E_r E_t G_0$. Data collected 7 December allowed calculation of $E_t G_0$ and the calculation of E_r's.

Collected data is presented in Appendix E.

1. Two Way Propagation Path Measurements

Two way propagation path data were collected 28 and 30 November. Measurements were used to calculate the product $\sigma_b E_r E_t G_0$.

Figure 8 provides a connection schematic for the data acquisiton equipment.

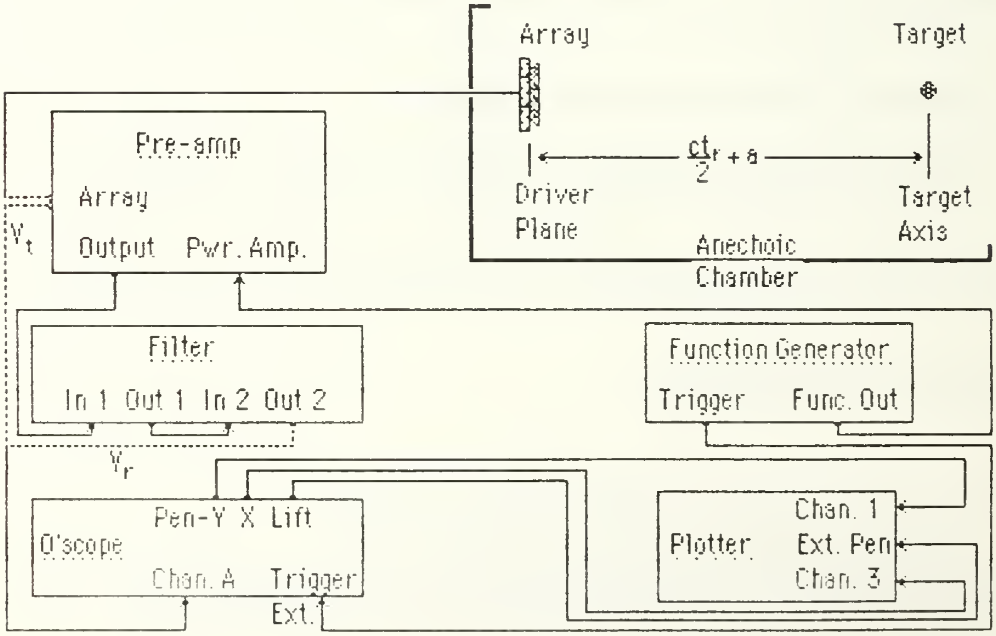


Figure 8: Data Acquisition Equipment Set-up

Four target spheres were available and measured for size. They included one hollow aluminum sphere (sphere number one) and three solid aluminum and brass spheres. Table (1) lists the data and the calculated values of $A_{tgt} = \pi a^2$, ka, and σ_b. σ_b was calculated from equation (86b).

Table 1: Target Sphere Dimensional Data and Normalized Backscattered Cross Section Calculational Results for Frequency of 5000 [Hz].

Target No.	Sphere Diameter [m]	Sphere Radius [m]	A_{tgt} [m^2]	ka	σ_b
1	.2546 ±.0006	.1273	.05091	11.62	.9142
2	.1013 ±.0001	.05065	.008060	4.619	1.000
3	.07634±.00003	.03817	.004577	3.479	1.051
4	.06240±.00008	.03120	.003058	2.843	.6836

Figure 9 shows the target spheres' theoretical normalized backscattered cross section distribution plotted on a graph generated by the backscattered cross section program of Appendix C.

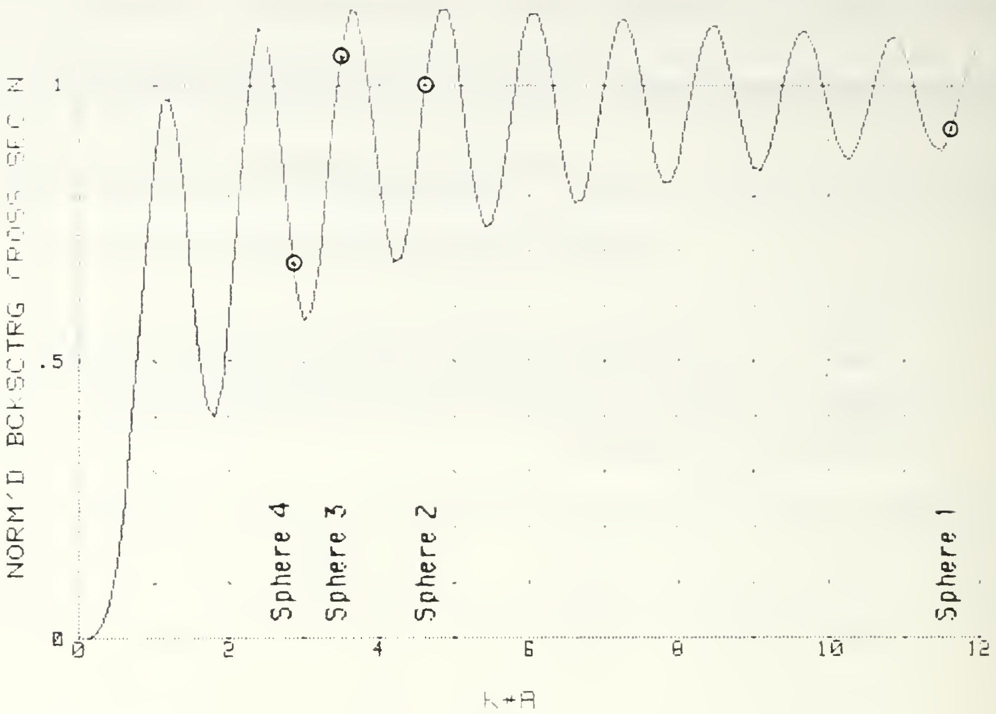


Figure 9: Target Spheres' Normalized Backscattered Cross Section Distribution as Calculated from Measured Diameters, Using an Assumption of Perfect Acoustic Hardness.

The target spheres were suspended in turn in the anechoic chamber. Placement was made using the criteria in Section D.1 of Chapter II. The range R from the array to a target was determined to be 5.30 [m].

Temperature T_c and relative humidity Rh along the propagation path were determined with a Weathermeasure Temperature and Relative

Humidity Meter. T_c was usually about 21 [°c] with a relative humidity of approximately 50%. Atmospheric pressure P was read from a barogram supplied by the Naval Postgraduate School meteorological station. Specific T_c, Rh, and P data are furnished in Appendix E.

The unshrouded, close packed, hexagonal acoustic array of Moxcey [Ref. 8] was used to ensonify each target.

$$A = N_s \pi r_s^2 \qquad (96)$$

determined the aperture area A of the array. $N_s = 19$ was the number of speakers in the array and $r_s = .0381$ [m] was the average speaker horn radius [Ref. 8].

Moxcey [Ref. 8] determined the half width of the array's main lobe to be 12°. The angular half widths of targets two, three and four were all less than 5% of the main lobe's half width. Target one's angular half width was 11.5% of the main lobe's half width.

A sinusoidal input from a Hewlett Packard 3314 A function generator drove the array via a pre-amplifier. Signal input from the HP3314A function generator to the pre-amplifier was set for 20 cycles at 5000 [Hz]. The voltage input to the pre-amplifier's power amplifier terminal (see Figure 8) was 3.5 [V] for spheres two, three and four and 3.0 [V] for sphere one.

Twenty cycles is considerably less than the number of cycles expected in actual echosounder application. It was necessary to keep the pulse packet this small because the anechoic chamber's corner was only about 1.5 [m] from the target mounting location. Voltage input for sphere one was reduced to 3.0 [V] to avoid clipping of its greater return signal voltage.

Actual transmission voltage inputs V_t to the array from the pre-amplifier were measured with a Nicolet 3091 oscilloscope at the pre-amplifier's array terminal. The array was disconnected from the pre-amplifier for V_t measurements. V_t values are provided in Appendix E. They are also stored in the Trans30 and Trans35 subprograms of the acoustic array efficiency program in Appendix D.

The Nicolet oscilloscope was connected to a Hewlett Packard 7090 A measurement plotting system. This connection provided a hard copy plot of voltage vs. time traces.

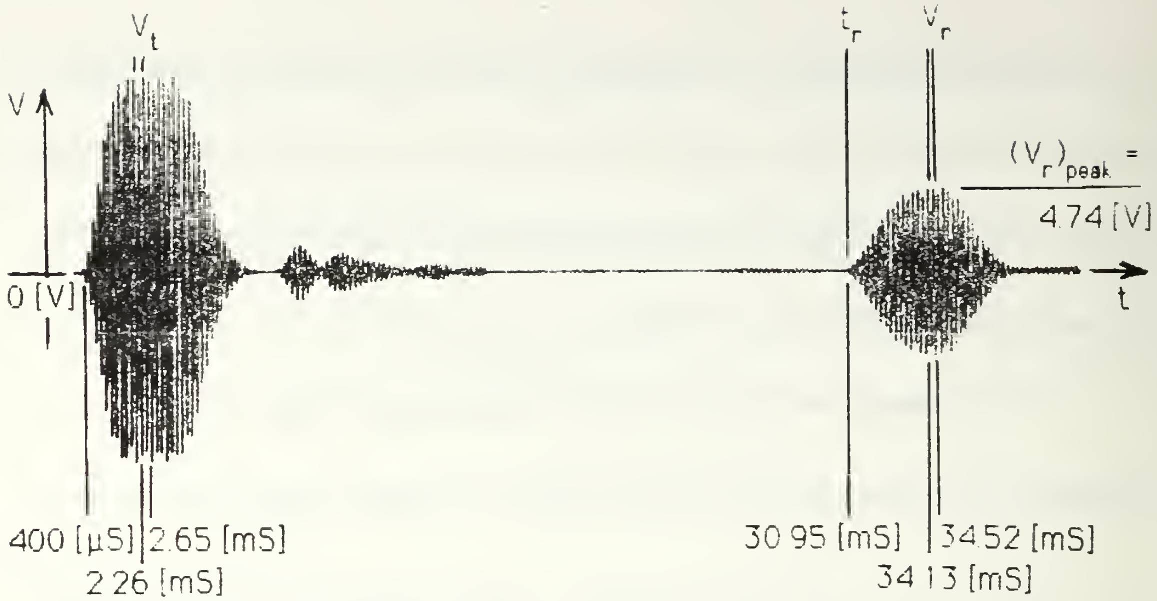
The returned signal was amplified and filtered. Pre-amplifier gain G_e was determined as 11,094 by Moxcey [Ref. 8].

Filtering was done with a Wavetek Brickwall filter model 753 A to remove the low frequency normal modes of oscillation of the speaker drivers from the return signal. Filtering was accomplished at 0 [dB] gain and 5000 [Hz] for both low and high pass settings.

The processed return signal voltages V_r were displayed and measured on the Nicolet 3091 oscilloscope. V_r measurements were made at the Out 2 terminal of the Wavetek Brickwall filter (see Figure 8). They were taken at 10 [µS] intervals for two full cycles along the asymptotic region of the return signal trace.

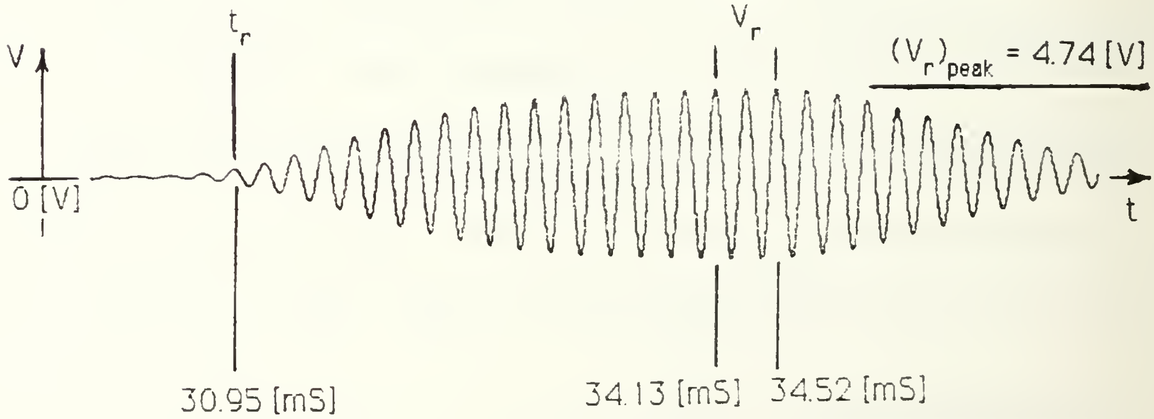
The Nicolet 3091 oscilloscope was connected to the Hewlett Packard 7090 A measurement plotting system. Figure 10 shows the Hewlett Packard plotter's traces for target sphere two's return signal voltages as printed and subsequently labeled. The figure is representative of the oscilloscope traces for each target return. Data for all targets are provided in Appendix E.

The time of return t_r was determined as the time difference between the first positive maxima of the transmitted pulse packet and the first positive maxima of the target's return pulse packet.



18: 27: 17 28 NOV 87

Figure 10, Sheet 1 of 2: Target Sphere Two's 5000 [Hz] Return Signal Trace



18: 28: 37 28 NOV 87

Figure 10, Sheet 2 of 2: Target Sphere Two's 5000 [Hz] Return Signal Trace, Scaled Down For Oscilloscope Measurement.

Noise voltage V_n measurements were made with the same measuring equipment setup used for V_r measurements. The target spheres were removed. V_n measurements were taken at a time of return t_r comparable to the time of measurement of V_r.

V_n measurements were actually made on occasions well separated chronologically from the V_r measurements. Therefore, they were proportionally corrected in line 2530 of the acoustic array efficiency program. The corrections account for the slight variations in input voltages and propagation path attenuations that occurred between the two measurement occasions.

Figure 11 shows a representative noise trace. Data from the noise measurements is presented in Appendix E and stored in the Noise30 and Noise35 subprograms of the acoustic array efficiency program in Appendix D.

Unless otherwise specified above the remaining data from the two way propagation path measurements are stored in the subprograms titled Sphere1, Sphere2, Sphere3 and Sphere4. All remaining data are also presented in Appendix E.

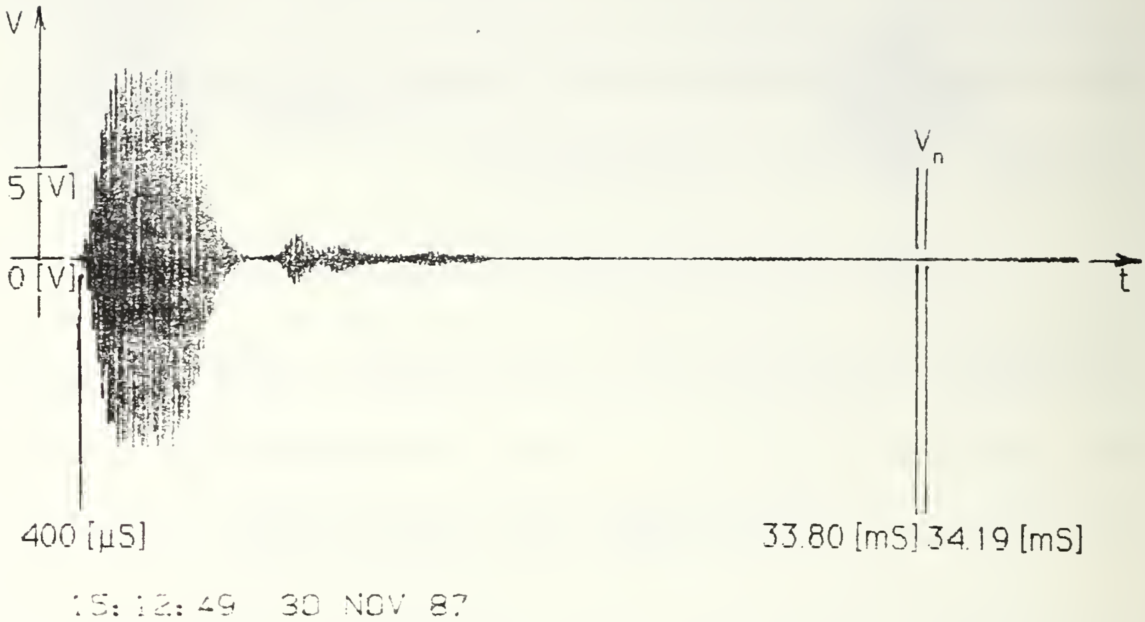


Figure 11: Representative 5000 [Hz] Noise Trace.
3.0 [V] Supplied by the HP3314A Function Generator.

2. One Way Propagation Path Measurements

One way propagation path data were collected 7 December. Measurements were used to calculate E_tG_o with equation (17).

Data from the one way propagation path measurements are provided in Appendix E. Data are also stored in the Gain30 and Gain35 subprograms of the acoustic array efficiency program of Appendix D.

For one way path measurements, i.e. for the determination of E_tG_0, an Ivie Electronics Inc. IE 30 A Audio Analyzer (Serial *805 A 426) was used to measure received target intensity I_r in [dB]. Intensity measurements were referenced to 10^{-12} [W/m^2]. The acoustic signal supplied for this measurement was a continuous wave at 5000 [Hz]. Measurements were taken at the suspension site of the calibrating targets.

A Fluke 8060 A True RMS Multimeter was used to measure the root-mean-square voltage $(V_{rms})_t$ supplied to the acoustic array at its input. $(V_{rms})_t$ was squared to determine $(V_{ms})_t$.

The Weathermeasure temperature and relative humidity meter was unavailable for E_tG_0 data collection. Hygrothermographs from the Naval Postgraduate School's meteorological station were used to estimate the anechoic chamber's relative humidity Rh and temperature T_c on 7 December. Estimates were based on comparisons to the hygrothermographs and measured Rh and T_c of 28 and 30 November.

C. RESULTS

Data analysis was performed with the aid of the acoustic array efficiency program of Appendix D. Data used in calculations is furnished in Appendix E.

The product $\sigma_b E_r E_t G_o$ was generated from two way popagation path data and application of equation (26). The average of calculated σ_b's compared to the average of the experimental product $\sigma_b E_r E_t G_o$ gives a normalizing product $E_r E_t G_o$. The experimental product $\sigma_b E_r E_t G_o$ and the normalizing product $E_r E_t G_o$ are used to calculate experimental σ_b's. These results are supplied in Table 2.

Table 2: Target Sphere Experimental, Normalized Backscattered Cross Section Calculational Results.

Target No.	ka [m^2]	Calc'd σ_b	$\sigma_b E_r E_t G_o$	$E_r E_t G_o$	Exp'l σ_b
1	11.62	.9142	55.90		.947
2	4.619	1.000	59.88		1.01
3	3.479	1.051	61.62		1.04
4	2.843	.6836	37.98		.644
Avg.		.912	53.85	59.0	

The experimental σ_b are plotted on a graph generated by the backscattered cross section program of Appendix C. This plot is Figure 12. It is similar to Figure 9 and gives σ_b as a function of ka. Note ka is labeled K*A.

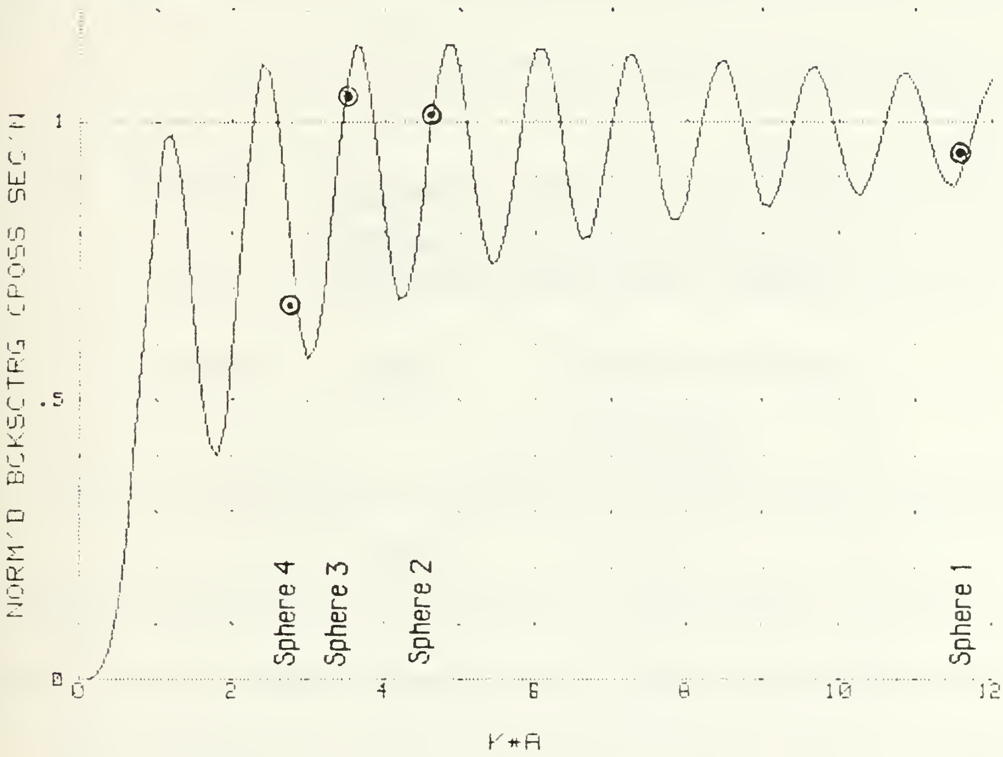


Figure 12: Target Spheres' Experimental, Normalized Backscattered Cross Section Distribution Compared to a Calculated Backscattered Cross Section Distribution Curve Generated by the Backscattered Cross Section Program of Appendix C.

As ka increases the experimental σ_b distribution rises from just below the calculated σ_b curve to just above it.

E_tG_o was determined using one way propagation path data and equation (17). E_tG_0, or E_t, was assumed to be the same for spheres two, three, and four because the supply voltage and target locations were the same. The angular half widths of target spheres two, three and four were between 4.6% and 2.8% of the echosounder's main lobe's half width.

Table 3: Acoustic Array Product E_tG_o Determined from One Way Propagation Path Measurements and Equation (17).

Target No.	HP3314A Func'n Gen'r Supply Volt. [V]	E_tG_o
1	3.0	94.70
2	3.5	95.11
3	3.5	95.11
4	3.5	95.11

Calculated σ_b, E_tG_o and two way propagation path data were used in equation (26) to determine E_r. σ_b values were calculated from the target spheres' measured diameters, 2a. A_{tgt} was calculated as πa^2. Calculation results are furnished in Table 4.

Table 4: E_r and E_tE_r. E_r Determined From Two Way Propagation Path Data and Equation (26). E_tE_r Determined From Assumption of Efficiency Reciprocity.

Target No.	A_{tgt} [m^2]	Calc'd σ_b	$\sigma_b A_{tgt}$ [m^2]	E_r	E_tE_r
1	.05091	.9142	.04654	.6456	.4169
2	.008060	1.000	.008060	.6296	.3964
3	.004577	1.051	.004810	.6167	.3803
4	.003058	.6836	.002090	.5841	.3411

The E_r's determined for Moxcey's unshrouded hexagonal array are in the range of .58 to .65. This is above the value range of 0.5 ± 0.1 determined by Weingartner for his square acoustic array [Ref. 6].

Table 4 results were used to construct Figures 13 and 14. Figure 13 gives E_r as a function of the calculated, normalized backscattered cross section, σ_b. Figure 14 gives E_r as a function of the apparent target size, $\sigma_b A_{tgt}$.

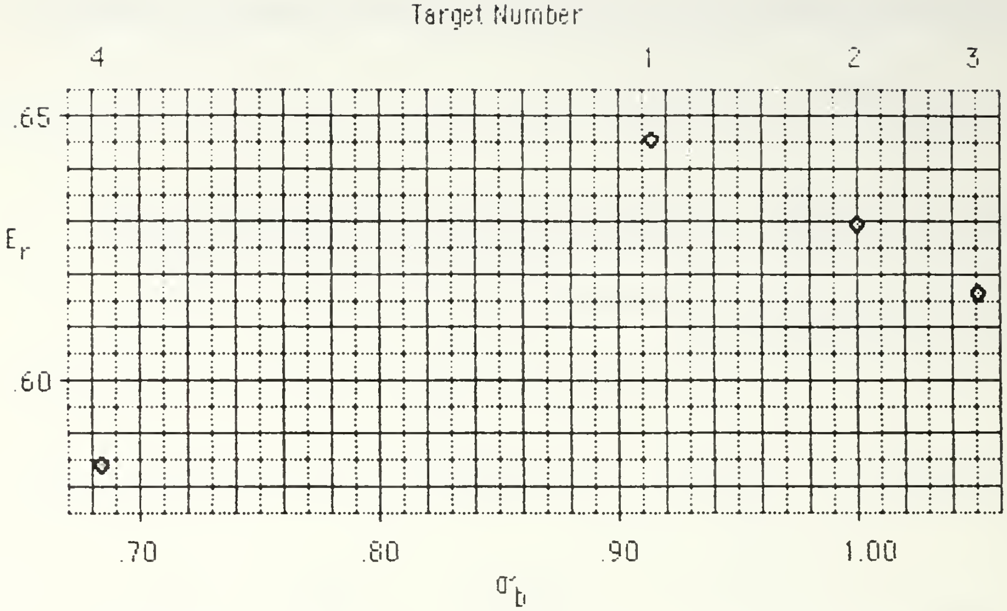


Figure 13: Acoustical to Electrical Power Conversion Efficiencies, $E_{r,}$ as a Function of the Normalized Backscattered Cross Sections, σ_b, Calculated from Measured Sphere Sizes.

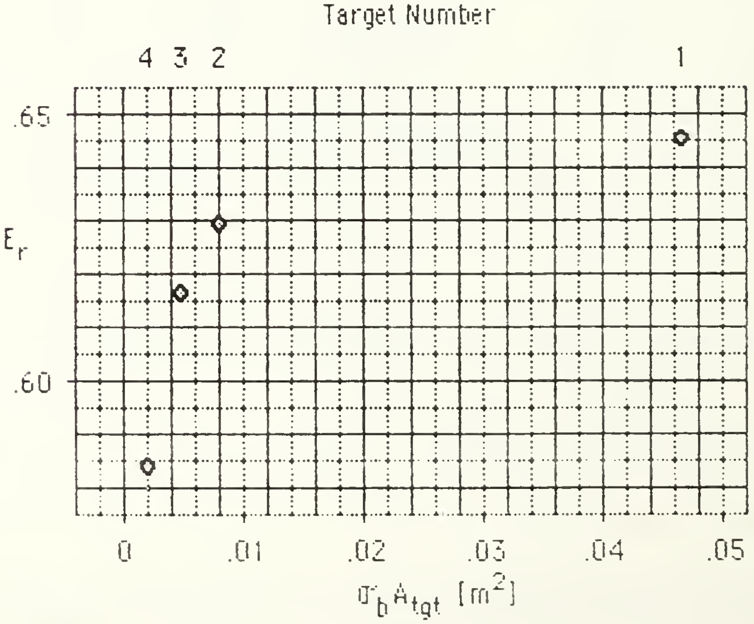


Figure 14: Acoustical to Electrical Power Conversion Efficiencies, $E_{r,}$ as a Function of the Apparent Target Sizes, $\sigma_b A_{tgt}$.

Recalling equation (20),

$$P'_{tgt} = \left[\frac{P_a}{4\pi} \right] \left[e^{-\alpha R} G(\Omega) \right] \left[\frac{A_{tgt}}{R^2} \right] , \qquad (20)$$

and equation (12),

$$P_{ra} = \frac{P_{tgt}}{4\pi R^2} \sigma_b e^{-\alpha R} A , \qquad (12)$$

serves as a reminder that the received acoustic power, P_{ra}, depends on the apparent target size, $\sigma_b A_{tgt}$.

Figure 14's trend and the dependency of P_{ra} on $\sigma_b A_{tgt}$ were used as suggestions for the construction of Figure 15. Figure 15 is a plot of E_r

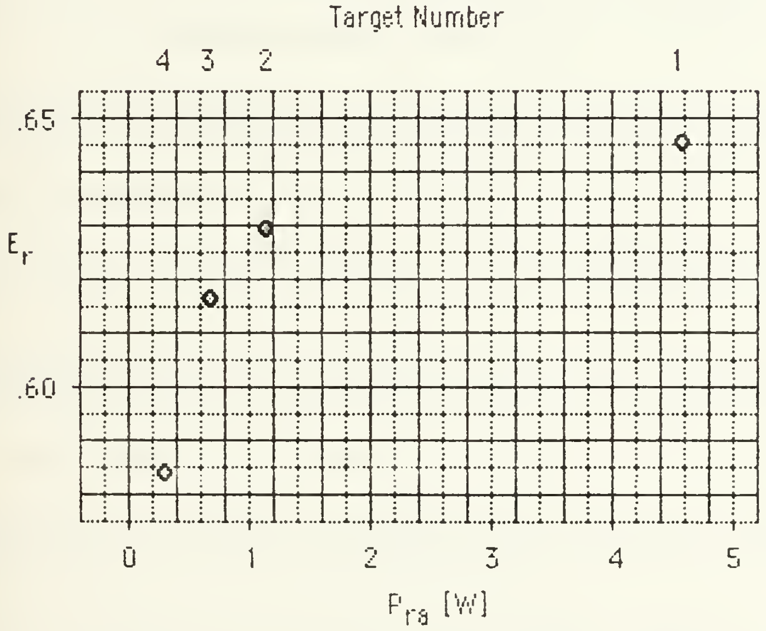


Figure 15: Acoustical to Electrical Power Conversion Efficiencies, E_r, as a Function of the Received Acoustic Powers, P_{ra}.

versus P_{ra}. P_{ra} was calculated using the definition (from equation (21))

$$P_{ra} = P_r / E_r \qquad (97)$$

and the relationship (from equation (15))

$$P_r = (V_{ms})_r / Z \ . \qquad (98)$$

Note Figure 15's trend is for E_r to increase with an increase in P_{ra} for the range of target sizes investigated.

IV. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The larger magnitudes of E_r determined for Moxcey's hexagonal array suggest it may be more efficient than Weingartner's square array. However, the drivers for each array are identical.

The 0.08 to 0.15 differences in efficiencies are more likely the result of the difference in calibration methods. Weingartner's measurements [Ref. 6] used steady state, continuous waves. The use of steady state, continuous waves allows the development of standing waves. This calibration investigation used pulsed waves. The use of pulsed waves did not provide sufficient time for standing waves to completely develop in the backscattered energy. [Ref. 26] This calibration method should provide more accurate efficiency values since it more nearly approximates the echosounder system's actual field operating configuration.

Additionally, steel braid and nylon lines were used to suspend the target spheres. These supports probably reflected enough acoustic energy to have a small influence on results. Other fixtures in the anechoic chamber, such as support clamps and lights, may have also contributed small amounts of reflected energy to the returned signal.

Calibration of the computer controlled echosounder required determination of the product $E_r E_t$. The calibration process assumed $E_r = E_t$. The calibration results in Tables 3 and 4 for spheres two, three, and four indicate E_r varied with constant $E_t G_0$, or constant E_t. Such a variation negates the assumption of reciprocity for E_r and E_t.

Figure 15 suggests the acoustic array's efficiency E_r increased in response to an increase in P_{ra} and $(V_{rms})_r$.

The acoustic gain for spheres two, three and four was probably not constant, as assumed. However, G_0 is the maximum for $G(\Omega)$, and E_r is inversely proportional to G_0, per equation (26). An averaging process accounting for the slight differences in gain encountered by spheres two, three and four would have increased the data points spread observed in Figures 14 and 15.

Additionally, the pre-amplifier's gain may not have been constant for the range of received voltage values encountered. This would have introduced either an apparent increase in E_r or dampened an increase in E_r

with an increase in $(V_{rms})_r$. Such a gain variation could have effectively negated the assumption of efficiency reciprocity. However, this pre-amplifier is approximately 0.1% linear for less than 10 [V] peak [Ref. 26]. Received voltages in this investigation were all less than 10 [V].

B. RECOMMENDATIONS

Because efficiency reciprocity is now in question it is necessary to determine G_0 by calculation or measurement. Such a determination would allow the desired calibration quantity $E_r E_t$ to be written as

$$E_r E_t = E_r \frac{E_t G_0}{G_0} . \qquad (99)$$

One method of calculating G_0 would involve integrating the acoustic array's calculated or measured intensity pattern (see Moxcey [Ref. 8]) for determination of an equivalent isotropic emitter. To determine G_0 the centroid intensity of the array's emitted intensity pattern would be compared to that of the equivalent isotropic emitter. This would be repeated for each transmission power level of interest.

Calculation of G_0 would allow a determination of E_t using E_tG_0. Such a determination of E_t would allow a reciprocity comparison to E_r and support a more accurate evaluation of E_rE_t.

The accuracy of E_r and E_tG_0 could be further refined as follows.

Use a calibrating site that allows the use of a pulse train closer in length to actual field use.

Make target return and noise voltages and site intensity measurements as chronologically close as possible.

Measure the barometric pressure in the propagation path directly along with the relative humidity and temperature.

The received and transmitted electrical powers for E_r and the transmitted electrical power for E_tG_0 were all measured at the pre-amplifier end of the pre-amplifier/array transmission line. This included transmission line power losses due to impedance as efficiency decreases. The present calibration process includes the transmission line impedance as part of the array impedance.

The amount this impedance power loss actually affects E_r could be evaluated by determining the impedances seen in each direction at the pre-amplifier's array terminal. This would be an opportune time to verify the validity of transmission line impedance reciprocity.

Use the Nicolet oscilloscope to record a triggered readout from a calibrated microphone suspended at the target site at 45^0 to the centroid's axis. This would allow E_tG_0 to be determined for a pulse packet rather than for a continuous wave. Coupled with a proper calibrating site selection this technique should minimize interference problems such as anechoic chamber corner reflections.

A larger number of apparent target sizes spanning the desired range of apparent target sizes should be used to redetermine E_r for Moxcey's array. The E_r distribution should be examined for a smooth and asymptotic behavior. Comparative E_r values for Moxcey's array could be determined using the method outlined by Weingartner [Ref. 6].

Weingartner's square array's E_r could also be determined with this thesis' calibration process and compared to Weingartner's E_r value [Ref. 6].

The refined E_r curve in the now calibrated echosounder system should be used to develop an absolute $C_T{}^2$ plot of an air mass. An established method such as a tower mounted vertical array of thermocouples would be used simultaneously to generate a comparative absolute C_T^2 plot of the same air mass.

C. SUMMARY

Realizing the potential of the computer controlled echosounder for analyzing lower atmospheric turbulence required a calibration of the echosounder. This thesis described theory and software for performing calculations crucial to a backscattered cross section calibration of the echosounder. The backscattered cross section calibration was evaluated in this thesis.

E_r's values indicate the evaluated calibration process requires some refinement. They also indicate the calibration method possesses sufficient merit to warrant further development and that the software performs as intended.

APPENDIX A

ANECHOIC CHAMBER DESCRIPTION

The anechoic chamber is located in room 019 of building 232 at the Naval Postgraduate School. Acoustic research on sound sources, sound receivers and sound scatterers may be conducted in the chamber with a minimum of interference from wall reflection and external noise.

Wall reflection is minimized by 102 centimeter deep wedges made of P.F. 612 fiberglass. The wedges are attached to the walls, ceiling and floor of the chamber. Approximately 142 cubic meters of fiberglass is used to absorb 99% of incident sound energy for frequencies greater than 100 Hertz. External noise isolation is accomplished by separating the chamber's inner concrete block sides from the outer 12 inch concrete walls with a two inch thick lining of fiberglass and cork.

A usable region of approximately 8.2 meters by 4.3 meters by 3.4 meters is available. This region is floored by a grid of 225 wire cables, with each cable placed in a tension of 150 to 200 pounds per square inch. *

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*This Appendix was compiled from the description posted by the entrance to the anechoic chamber instrumentation and control room.

APPENDIX B

DIFFERENTIAL SCATTERING CROSS SECTION PROGRAM

Examples of the output of this program are Figures 4 and 5, located in Chapter III.

```
10    !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
20    !
30    REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
40    REM      ACOUSTICALLY HARD SPHERE SCATTERING OF PLANE ACOUSTIC WAVES
50    REM
60    REM      THIS PROGRAM CALCULATES AND GRAPHS THE DIFFERENTIAL SCATTERING
70    REM   CROSS SECTION AND/OR SCATTERING MODULUS FOR AN ACOUSTICALLY HARD
80    REM   SPHERE OF DIAMETER D ENSONIFIED BY AN INCIDENT PLANE WAVE OF
90    REM   FREQUENCY F.
100   REM
130   REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
140   !
150   !++++ VARIABLE DECLARATIONS AND DEFINITIONS +++++++++++++++++++++++++++++
160                                       !
170                                       !
180   REAL C                              ! SPEED OF SOUND.
190                                       !
200   REAL Freq                           ! FREQUENCY OF THE EMITTER.
210                                       !
220   REAL A                              ! RADIUS OF THE SPHERE.
230                                       !
240   REAL L                              ! ORDER OF THE FUNCTION CALCULATED
250                                       ! IN A SUBROUTINE.  IT IS USED
260                                       ! AS A LOOP INDEX AND TO DESIGNATE
270                                       ! THE ARRAY ELEMENT FOR THE FUNCTION
280                                       ! OF CORRESPONDING ORDER.
290                                       !
300   REAL Lmax                           ! MAX. SIGNIFICANT ORDER OF THE SUM
310                                       ! DETERMINING THE DIFFERENTIAL
320                                       ! SCATTERING CROSS SECTION (DSCS)
330                                       ! AND SCATTERING MODULUS (SM).
340                                       !
350   REAL K                              ! THE WAVE NUMBER, 2*PI*FREQ/C.
360                                       !
370   REAL Ka                             ! K*A, THE ARGUMENT OF THE REGULAR
380                                       ! AND IRREGULAR SPHERICAL BESSEL
390                                       ! FUNCTIONS AND THEIR DERIVATIVES.
400                                       !
```

```
410    REAL J(51)                          ! ARRAY FOR THE REGULAR SPHERICAL
420                                        ! BESSEL FUNCTIONS (RSB) OF
430                                        ! ARGUMENT KA AND ORDER 0 THROUGH
440                                        ! LMAX.
450                                        !
460    REAL Dj(51)                         ! ARRAY FOR THE DERIVATIVES OF THE
470                                        ! RSB (THE DRSB).
480                                        !
490    REAL Y(51)                          ! ARRAY FOR THE IRREGULAR SPHERICAL
500                                        ! BESSEL FUNCTIONS (ISB) OF
510                                        ! ARGUMENT KA AND ORDER 0 THROUGH
520                                        ! LMAX.
530                                        !
540    REAL Dy(51)                         ! ARRAY FOR THE DERIVATIVES OF THE
550                                        ! ISB (THE DISB).
560                                        !
570    REAL P(51)                          ! ARRAY FOR THE LEGENDRE
580                                        ! POLYNOMIALS OF ARGUMENT COS(PHI).
590                                        !
600    REAL Sm(360)                        ! THE ARRAY REPRESENTING THE
610                                        ! SCATTERING MODULUS (SM).
620                                        !
630    REAL Ds(360)                        ! THE ARRAY REPRESENTING THE
640                                        ! DIFFERENTIAL SCATTERING CROSS
650                                        ! SECTION NORMALIZED BY THE CROSS
660                                        ! SECTIONAL AREA OF THE SPHERE
670                                        ! (DSCS).
680                                        ! NOTE DS(PHI)=4*SM(PHI)^2/KA^2.
690                                        !
700                                        ! ++++ NOTE ++++
710                                        ! THE ARRAY DIMENSIONS (30+21) NEED
720                                        ! TO BE INCREASED TO ACCOMODATE
730                                        ! KA>30.
740                                        !
750    !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
760    !
770    !++++ MAIN PROGRAM +++++++++++++++++++++++++++++++++++++++++++++++++++++
780                                        !
790                                        !
800    CALL Init(C,Freq,A,Lmax,Ka)         ! INPUT AND CALCULATE REQUIRED
810                                        ! PARAMETERS.
820                                        !
830    CALL Difsctr(Lmax,Ka,Dj(*),J(*),Dy(*),Y(*),P(*),Sm(*),Ds(*))
840                                        ! CALCULATE THE SM(*) AND DS(*).
850                                        !
860    CALL Outpt(C,Freq,A,Ka,Sm(*),Ds(*)) ! OUTPUT THE SM(*) AND/OR DS(*).
870                                        !
880                                        !
890    !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
900    !
```

```
910    !++++ MAIN PROGRAM CONCLUSION ++++++++++++++++++++++++++++++++++++++++++
920                                        !
930    PRINTER IS 1
940    PRINT
950    PRINT "END"                         ! ADVISES THE USER OF THE MAIN
960    PRINT                               ! PROGRAM'S CONCLUSION.
970    END
980                                        !
990    !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1000   !
1010   !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
1020   !
```

```
1030  !
1040  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
1050  SUB Init(C,Freq,A,Lmax,Ka)
1060  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1070  REM   THIS MODULE REQUESTS THE APPROPRIATE INPUT VARIABLES TO DETERMINE
1080  REM   C, FREQ, A, LMAX, AND KA.
1090  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1100  !
1110  !++++ MODULE VARIABLE DECLARATIONS AND DEFINITIONS +++++++++++++++++++++
1120                                      !
1130                                      !
1140  REAL Temp                           ! AMBIENT AIR TEMPERATURE IN DEG C.
1150                                      !
1160  REAL Circ                           ! CIRCUMFERENCE OF THE TGT. SPHERE.
1170                                      !
1180  REAL Diam                           ! DIAMETER OF THE TARGET SPHERE.
1190                                      !
1200                                      !
1210  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1220  !
1230  !++++ REQUESTING AND CALCULATING SYSTEM PARAMETERS ++++++++++++++++++++++
1240                                      !
1250                                      !
1260  PRINTER IS 1                        ! INPUTS ARE REQUESTED ON THE CRT.
1270                                      !
1280  Choice=0                            ! *** DETERMINING C ***
1290  WHILE Choice<>1 AND Choice<>2
1300    PRINT
1310    PRINT "DO YOU DESIRE TO INPUT"
1320    PRINT "     (1) THE SPEED OF ";
1330    PRINT "SOUND,"
1340    PRINT "  OR (2) THE AMBIENT AIR ";
1350    PRINT "TEMPERATURE?"
1360    PRINT
1370    INPUT Choice
1380  END WHILE
1390                                      !
1400  IF Choice=1 THEN                    ! DIRECTLY ENTER C,
1410    PRINT "PLEASE ENTER C IN M/S."
1420    INPUT C
1430  END IF
1440                                      !
1450  IF Choice=2 THEN                    ! OR CALCULATE C FROM TEMP.
1460    PRINT "PLEASE ENTER TEMP IN ";
1470    PRINT "DEGREES CELSIUS."
1480    INPUT Temp
1490    Temp=Temp+273.15
1500    C=20.05*SQR(Temp)
1510  END IF
1520                                      ! *** DETERMINING FREQ ***
1530  PRINT                               ! DIRECTLY ENTER FREQ.
1540  PRINT "PLEASE ENTER THE ECHOSOUNDER";
1550  PRINT " FREQUENCY IN HZ."
1560  PRINT
1570  INPUT Freq
```

```
1580                                        !
1590  Choice=0                              ! *** DETERMINING KA ***
1600  WHILE Choice<>1 AND Choice<>2 AND Choice<>3
1610    PRINT
1620    PRINT "DO YOU DESIRE TO INPUT "
1630    PRINT "      (1)THE SPHERICAL ";
1640    PRINT "BESSEL ARGUMENT K*A,"
1650    PRINT "      (2) THE SPHERE'S ";
1660    PRINT "CIRCUMFERENCE,"
1670    PRINT "   OR (3) THE SPHERE'S ";
1680    PRINT "DIAMETER?"
1690    PRINT
1700    INPUT Choice
1710  END WHILE
1720                                        !
1730  IF Choice=1 THEN                      ! DIRECTLY ENTER KA,
1740    PRINT "PLEASE ENTER K*A."
1750    INPUT Ka
1760  END IF
1770                                        !
1780  IF Choice=2 THEN                      ! CALCULATE KA FROM THE
1790    PRINT "PLEASE ENTER THE TARGET "; ! TARGET CIRCUMFERENCE,
1800    PRINT "CIRCUMFERENCE IN CM."
1810    INPUT Circ
1820    Ka=Circ*(10^(-2))*Freq/C
1830  END IF
1840                                        !
1850  IF Choice=3 THEN                      ! OR CALCULATE KA FROM THE
1860    PRINT "PLEASE ENTER THE TARGET "; ! TARGET DIAMETER.
1870    PRINT "DIAMETER IN CM."
1880    INPUT Diam
1890    Ka=Diam*(10^(-2))*PI*Freq/C
1900  END IF
1910                                        !
1920                                        ! *** DETERMINING REMAINING ***
1930                                        ! *** SYSTEM PARAMETERS     ***
1940  K=2*PI*Freq/C                         ! CALCULATE THE WAVE NUMBER.
1950  A=Ka/K                                ! CALCULATE THE SPHERE RADIUS.
1960  Lmax=INT(Ka+21)                       ! MAX. SUMMATION ORDER CALCULATED.
1970                                        !
1980                                        !
1990  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
2000  !
2010  !++++ MODULE CONCLUSION +++++++++++++++++++++++++++++++++++++++++++++++
2020                                        !
2030  SUBEND
2040                                        !
2050  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
2060  !
2070  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
2080  !
```

```
2090  !
2100  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
2110  SUB Difsctr(Lmax,Ka,Dj(*),J(*),Dy(*),Y(*),P(*),Sm(*),Ds(*))
2120  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
2130  REM   THIS MODULE CALCULATES THE ARRAYS SM(*) AND DS(*) REPRESENTING THE
2140  REM   SCATTERING MODULUS AND THE DIFFERENTIAL SCATTERING CROSS SECTION
2150  REM   NORMALIZED BY THE CROSS SECTIONAL AREA OF THE SPHERE, RESPECTIVELY.
2160  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
2170  !
2180  !++++ MODULE VARIABLE DECLARATIONS AND DEFINITIONS ++++++++++++++++++++++
2190                                       !
2200                                       !
2210  REAL Fac                             ! FACTOR COMMON TO RSM AND ISM.
2220                                       !
2230  REAL Rsm                             ! THE REAL COMPONENT OF THE
2240                                       ! SCATTERING MODULUS, SM.
2250                                       !
2260  REAL Ism                             ! THE IMAGINARY COMPONENT OF THE
2270                                       ! SM.
2280                                       !
2290  REAL Fact                            ! FACTOR COMMON TO SM(PHI) & DS(PHI).
2300                                       !
2310  REAL Phi                             ! THE (POLAR) ANGLE OFF THE AXIS
2320                                       ! OF PROPAGATION OF THE INCIDENT
2330                                       ! PLANE WAVE, WITH ORIGIN AT THE
2340                                       ! SPHERE CENTER.  COS(PHI) IS THE
2350                                       ! ARGUMENT OF THE LEGENDRE
2360                                       ! POLYNOMIALS P(*) OF ORDER 0
2370                                       ! THROUGH LMAX.
2380                                       !
2390                                       !
2400  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
2410  !
2420  !++++ CALCULATING THE SM AND DSCS ARRAYS ++++++++++++++++++++++++++++++++
2430                                       !
2440                                       !
2450  CALL Drsb(Lmax,Ka,Dj(*),J(*))        ! CALCULATE THE DERIVATIVES OF THE
2460                                       ! REGULAR SPHERICAL BESSEL FUNCTIONS
2470                                       ! OF ORDER 0 THROUGH LMAX.
2480  CALL Disb(Lmax,Ka,Dy(*),Y(*))        ! CALCULATE THE DERIVATIVES OF THE
2490                                       ! IRREGULAR SPHERICAL BESSEL FUNC'NS
2500                                       ! OF ORDER 0 THROUGH LMAX.
2510  FOR Phi=0 TO 360
2520                                       !
2530    CALL Leg(Lmax,Ka,Phi,P(*))         ! CALCULATE THE ARRAY P(*).
2540                                       !
2550    Rsm=0                              ! CALCULATING THE REAL AND IMAGINARY
2560    Ism=0                              ! COMPONENTS OF THE SM.
2570    FOR L=0 TO Lmax
2580      Fac=(Dj(L)*(2*L+1)*P(L))/(Dj(L)*Dj(L)+Dy(L)*Dy(L))
2590      Rsm=Rsm+(Fac*Dy(L))
2600      Ism=Ism+(Fac*Dj(L))
2610    NEXT L
```

```
2620                                      !
2630    Fact=Rsm*Rsm+Ism*Ism
2640                                      !
2650    Sm(Phi)=SQR(Fact)                 ! CALCULATING THE ARRAY REPRESENTING
2660                                      ! THE SM(PHI).
2670    Ds(Phi)=4*(Fact)/(Ka*Ka)          ! CALCULATING THE ARRAY REPRESENTING
2680                                      ! THE DIFFERENTIAL SCATTERING CROSS
2690                                      ! SECTION NORMALIZED BY THE CROSS
2700                                      ! SECTIONAL AREA OF THE SPHERE.
2710  NEXT Phi
2720                                      !
2730                                      !
2740  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
2750  !
2760  !++++ MODULE CONCLUSION +++++++++++++++++++++++++++++++++++++++++++++++++
2770                                      !
2780  SUBEND
2790                                      !
2800  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
2810  !
2820  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
2830  !
```

```
2840  !
2850  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
2860  SUB Drsb(Lmax,Ka,Dj(*),J(*))
2870  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
2880  REM   THIS MODULE CALCULATES THE DERIVATIVES OF THE REGULAR SPHERICAL
2890  REM   BESSEL FUNCTIONS (DRSB) OF ARGUMENT KA AND ORDERS 0 THROUGH LMAX.
2900  REM   THE CALCULATION OF THE DERIVATIVE INVOLVES THE RSB'S OF THE SAME
2910  REM   AND PREVIOUS ORDER.
2920  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
2930  !
2940  !++++ CALCULATING THE DERIVATIVES OF THE RSB +++++++++++++++++++++++++++
2950                                        !
2960                                        !
2970  CALL Rsb(Lmax,Ka,J(*))                ! CALCULATE THE RSB VALUES J(*).
2980                                        !
2990  RAD                                   ! ANGLE UNITS FOR DRSB CALC'NS.
3000  Dj(0)=(-J(0)+COS(Ka))/Ka              ! CALCULATE THE INITIAL DRSB DJ(0).
3010  FOR L=1 TO Lmax                       ! CALCULATE THE REMAINING DRSB.
3020    Dj(L)=-((L+1)/Ka)*J(L)+J(L-1)
3030  NEXT L
3040                                        !
3050                                        !
3060  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3070  !
3080  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
3090                                        !
3100  SUBEND
3110                                        !
3120  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3130  !
3140  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
3150  !
```

```
3160 !
3170 !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
3180 SUB Rsb(Lmax,Ka,J(*))
3190 REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3200 REM   THIS MODULE CALCULATES THE REGULAR SPHERICAL BESSEL FUNCTIONS
3210 REM   (RSB) OF ARGUMENT KA AND ORDERS 0 THROUGH LMAX.   THE
3220 REM   CALCULATION USES THE CONTINUED FRACTION APPROACH.
3230 REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3240 !
3250 !++++ MODULE VARIABLE DECLARATIONS AND DEFINITIONS +++++++++++++++++++++
3260                                     !
3270                                     !
3280 REAL Rl                             ! THE EVOLVING RATIO OF J(0)/J(L).
3290                                     !
3300 REAL Numr                           ! THE EVOLVING NUMERATOR FACTOR USED
3310                                     ! IN CALCULATION OF RL.
3320                                     !
3330 REAL Denr                           ! THE EVOLVING DENOMINATOR FACTOR
3340                                     ! USED IN CALCULATION OF RL.
3350                                     !
3360 REAL Scexp                          ! EVOLVING SCALING EXPONENT FOR VERY
3370                                     ! LARGE OR VERY SMALL VALUES OF RL.
3380                                     !
3390 REAL Nflag                          ! 0 FLAG FOR NUMR.
3400                                     !
3410 REAL Dflag                          ! 0 FLAG FOR DENR.
3420                                     !
3430 REAL Bnm                            ! EVOLVING TERM USED IN THE
3440                                     ! CALCULATION OF NUMR AND DENR.
3450                                     !
3460 REAL Binc                           ! INCREMENT USED IN THE EVOLUTION
3470                                     ! OF BNM.
3480                                     !
3490                                     !
3500 !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3510 !
3520 !++++ CALCULATING THE RSB ++++++++++++++++++++++++++++++++++++++++++++++
3530                                     !
3540                                     !
3550 RAD                                 ! ANGLE UNITS FOR CALC'NS.
3560                                     !
3570                                     ! *** CALCULATE THE INITIAL ***
3580 J(0)=(SIN(Ka))/Ka                   ! *** RSB, J(0).            ***
3590                                     !
3600                                     ! *** CALCULATING THE RATIO ***
3610 FOR L=1 TO Lmax                     ! *** J(0)/J(L).            ***
3620   Rl=1.0                            ! INITIALIZING RL.
3630   Numr=0.                           ! INITIALIZING NUMR.
3640   Denr=0.                           ! INITIALIZING DENR.
3650   Scexp=0.                          ! INITIALIZING SCEXP.
3660   Nflag=0.                          ! INITIALIZING NFLAG.
3670   Dflag=0.                          ! INITIALIZING DFLAG.
3680   Bnm=1.0/Ka                        ! INITIALIZING BNM.
3690   Binc=2.0/Ka                       ! CALCULATE BINC.
3700                                     !
```

```
3710                                      ! * CALCULATE THE L "NAKED"     *
3720    FOR Ordcnt=1 TO L                 ! * NUMR FACTORS OF J(0)/J(L). *
3730      Bnm=Bnm+Binc                    ! INCREMENT BNM.
3740      IF Nflag=0 THEN                 ! CHECK NUMR=0 FLAG NOT SET;
3750        Numr=Bnm-Numr                 ! UPDATE NUMR IF NUMR<>0.
3760        IF Numr<>0 THEN
3770          Rl=Rl*Numr                  ! UPDATE RL IF UPDATED NUMR<>0.
3780          Numr=1.0/Numr               ! PREPARE NUMR FOR NEXT EVOLUTION.
3790                                      ! CHECK SCALING OF UPDATED RL.
3800          IF ABS(Rl)>10^250 THEN CALL Scale(Rl,Scexp)
3810        ELSE
3820          Nflag=1
3830        END IF
3840      ELSE                            ! IF NFLAG<>0 THE PREVIOUS STEPS ARE
3850        Nflag=0                       ! DEFERRED FOR ONE INCREMENT OF BNM.
3860      END IF
3870    NEXT Ordcnt
3880                                      ! * CALCULATE THE REMAINING NUMR *
3890                                      ! * AND DENR TERMS OF J(0)/J(L). *
3900                                      !   NOTE WHEN NUMR=DENR THEN
3910                                      !   RL=J(0)/J(L).
3920    WHILE Numr<>Denr OR Nflag=1 OR Dflag=1
3930      Bnm=Bnm+Binc                    ! INCREMENT BNM.
3940                                      !
3950      IF Nflag=0 THEN                 ! CHECK NUMR=0 FLAG NOT SET;
3960        Numr=Bnm-Numr                 ! UPDATE NUMR IF NUMR<>0.
3970        IF Numr<>0 THEN
3980          Rl=Rl*Numr                  ! UPDATE RL IF UPDATED NUMR<>0.
3990          Numr=1.0/Numr               ! PREPARE NUMR FOR NEXT EVOLUTION.
4000                                      ! CHECK SCALING OF UPDATED RL.
4010          IF ABS(Rl)>10^250 THEN CALL Scale(Rl,Scexp)
4020        ELSE
4030          Nflag=1
4040        END IF
4050      ELSE                            ! IF NFLAG<>0 THE PREVIOUS STEPS ARE
4060        Nflag=0                       ! DEFERRED FOR ONE INCREMENT OF BNM.
4070      END IF
4080                                      !
4090      IF Dflag=0 THEN                 ! CHECK DENR=0 FLAG NOT SET;
4100        Denr=Bnm-Denr                 ! UPDATE DENR IF DENR<>0.
4110        IF Denr<>0 THEN
4120          Rl=Rl/Denr                  ! UPDATE RL IF UPDATED DENR<>0.
4130          Denr=1.0/Denr               ! PREPARE DENR FOR NEXT EVOLUTION.
4140                                      ! CHECK SCALING OF UPDATED RL.
4150          IF ABS(Rl)>10^250 THEN CALL Scale(Rl,Scexp)
4160        ELSE
4170          Dflag=1
4180        END IF
4190      ELSE                            ! IF DFLAG<>0 THE PREVIOUS STEPS ARE
4200        Dflag=0                       ! DEFERRED FOR ONE INCREMENT OF BNM.
4210      END IF
4220                                      !
4230    END WHILE
4240                                      !
```

```
4250                                          !
4260    J(L)=J(0)/(R1*10^(Scexp))             ! *** J(L) DETERMINED. ***
4270                                          !
4280  NEXT L
4290                                          !
4300                                          !
4310  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4320  !
4330  !++++ MODULE CONCLUSION +++++++++++++++++++++++++++++++++++++++++++++++++++
4340                                          !
4350  SUBEND
4360                                          !
4370  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4380  !
4390  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
4400  !
```

```
4410  !
4420  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
4430  SUB Scale(Rl,Scexp)
4440  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4450  REM   THIS MODULE SCALES RL TO MAINTAIN NUMERICAL ACCURACY IN THE
4460  REM   ONGOING CALCULATION OF RL.
4470  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4480  !
4490  !++++ SCALING RL ++++++++++++++++++++++++++++++++++++++++++++++++++++++
4500                                     !
4510                                     !
4520  IF Rl>10^250 THEN
4530    Rl=Rl*10^(-250)
4540    Scexp=Scexp+250
4550  END IF
4560  IF Rl<10^(-250) THEN
4570    Rl=Rl*10^250
4580    Scexp=Scexp-250
4590  END IF
4600                                     !
4610                                     !
4620  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4630  !
4640  !++++ MODULE CONCLUSION +++++++++++++++++++++++++++++++++++++++++++++++
4650                                     !
4660  SUBEND
4670                                     !
4680  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4690  !
4700  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
4710  !
```

```
4720  !
4730  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
4740  SUB Disb(Lmax,Ka,Dy(*),Y(*))
4750  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4760  REM   THIS MODULE CALCULATES THE DERIVATIVES OF THE IRREGULAR SPHERICAL
4770  REM   BESSEL FUNCTIONS OF ARGUMENT KA AND ORDERS 0 THROUGH LMAX.  THE
4780  REM   CALCULATION OF THE DERIVATIVE USES THE ISB'S OF THE SAME AND
4790  REM   PREVIOUS ORDER.
4800  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4810  !
4820  !++++ CALCULATING THE DERIVATIVES OF THE ISB +++++++++++++++++++++++++++
4830                                      !
4840                                      !
4850  CALL Isb(Lmax,Ka,Y(*))              ! CALCULATE THE ISB VALUES.
4860                                      !
4870  RAD                                 ! ANGLE UNITS FOR CALC'NS.
4880  Dy(0)=(-Y(0)+SIN(Ka))/Ka            ! THE INITIAL ISB DERIVATIVE.
4890  FOR L=1 TO Lmax                     ! THE REMAINING ISB DERIVATIVES.
4900    Dy(L)=-((L+1)/Ka)*Y(L)+Y(L-1)
4910  NEXT L
4920                                      !
4930                                      !
4940  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4950  !
4960  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
4970                                      !
4980  SUBEND
4990                                      !
5000  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5010  !
5020  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
5030  !
```

```
5040  !
5050  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
5060  SUB Isb(Lmax,Ka,Y(*))
5070  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5080  REM   THIS MODULE CALCULATES THE IRREGULAR SPHERICAL BESSEL FUNCTIONS
5090  REM   OF ARGUMENT KA AND  ORDER L THROUGH LMAX.   THE CALCULATION IS
5100  REM   ACCOMPLISHED BY FORWARD RECURSION.
5110  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5120  !
5130  !++++ CALCULATING THE ISB ++++++++++++++++++++++++++++++++++++++++++++++
5140                                      !
5150                                      !
5160  RAD                                 ! ANGLE UNITS FOR CALC'NS.
5170  Y(0)=-COS(Ka)/Ka                    ! INITIAL ISB'S CALCULATED.
5180  Y(1)=(Y(0)-SIN(Ka))/Ka
5190  FOR L=2 TO Lmax                     ! REMAINING ISB'S CALC'D.
5200    Y(L)=((2*L-1)/Ka)*Y(L-1)-Y(L-2)
5210  NEXT L
5220                                      !
5230                                      !
5240  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5250  !
5260  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
5270                                      !
5280  SUBEND
5290                                      !
5300  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5310  !
5320  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
5330  !
```

```
5340  !
5350  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
5360  SUB Leg(Lmax,Ka,Phi,P(*))
5370  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5380  REM   THIS MODULE CALCULATES THE LEGENDRE POLYNOMIALS OF ARGUMENT
5390  REM   COS(PHI) AND ORDERS 0 THROUGH LMAX.  CALCULATION IS ACCOMPLISHED
5400  REM   BY FORWARD RECURSION.
5410  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5420  !
5430  !++++ CALCULATING THE LEGENDRE POLYNOMIALS +++++++++++++++++++++++++++++
5440                                      !
5450                                      !
5460  DEG                                 ! ANGLE UNITS FOR CALC'NS.
5470  X=COS(Phi)                          ! ARGUMENT OF THE LEG. POL'S.
5480                                      !
5490  P(0)=1                              ! INITIAL LEG. POL'S. CALC'D.
5500  P(1)=X
5510  FOR L=2 TO Lmax                     ! REMAINING LEG. POL'S. CALC'D.
5520    P(L)=(2-(1/L))*X*P(L-1)-(1-(1/L))*P(L-2)
5530  NEXT L
5540                                      !
5550                                      !
5560  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5570  !
5580  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
5590                                      !
5600  SUBEND
5610                                      !
5620  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5630  !
5640  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
5650  !
```

```
5660  !
5670  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
5680  SUB Outpt(C,Freq,A,Ka,Sm(*),Ds(*))
5690  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5700  REM   THIS MODULE CONTROLS THE OUTPUT OF THE CALCULATED DSCS AND SM IN
5710  REM   GRAPHICAL PRESENTATIONS.
5720  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5730  !
5740  !++++ MODULE VARIABLE DECLARATIONS AND DEFINITIONS +++++++++++++++++++++
5750                                       !
5760  REAL Desire                          ! INTERACTIVE LOOP CONTROL.
5770                                       !
5780  REAL Rpt                             ! POLARPLT SCALING CONTROL; A
5790                                       ! REPETITION INDEX.
5800                                       !
5810  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5820  !
5830  !++++ DETERMINING THE DESIRED OUTPUT ++++++++++++++++++++++++++++++++++++
5840                                       !
5850  Desire=0
5860  WHILE Desire<>1 AND Desire<>2 AND Desire<>3
5870    PRINT "DO YOU DESIRE"
5880    PRINT "     (1) A PLOT OF THE DSCS,"
5890    PRINT "     (2) A PLOT OF THE SM,"
5900    PRINT "  OR (3) BOTH?"
5910    INPUT Desire
5920  END WHILE
5930  PRINTER IS 701
5940                                       !
5950  WHILE Desire<4
5960    IF Desire=1 OR Desire=3 THEN
5970      FOR Rpt=1 TO 2
5980        CALL Polarplt(Ds(*),Rpt)       ! PLOTTING THE DSCS.
5990        PRINT
6000      NEXT Rpt
6010    END IF
6020                                       !
6030    IF Desire=2 THEN
6040      CALL Recplt(Sm(*))               ! PLOTTING THE SM.
6050      PRINT
6060    END IF
6070                                       !
6080    PRINT " ","C =";C;"M/S."           ! ECHO SIGNIFICANT CALCULATED VALUES
6090    PRINT " ","F =";Freq;"HZ."         ! AND INPUT PARAMETERS.
6100    PRINT " ","A =";A;"M."
6110    PRINT " ","K*A =";Ka;"."
6120    PRINT " ","NORMALIZED ";
6130    PRINT "BCKSCTR = ";Ds(180)
6140    PRINT CHR$(12)                     ! PRINTER FORM FEED.
6150                                       !
6160    IF Desire=1 OR Desire=2 THEN Desire=4
6170    IF Desire=3 THEN Desire=2
6180  END WHILE
6190                                       !
```

```
6200                                    !
6210  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
6220  !
6230  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
6240                                    !
6250  SUBEND
6260                                    !
6270  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
6280  !
6290  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
6300  !
```

```
6310  !
6320  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
6330  SUB Polarplt(Ds(*),Rpt)
6340  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
6350  REM   THIS MODULE MAPS THE CALCULATED, NORMALIZED DSCS ARRAY DS(*) ON
6360  REM   LINEAR POLAR PLOTS. THE FIRST PLOT INCLUDES THE ENTIRE DSCS,
6370  REM   WHILE THE SECOND PLOT SHOWS THE SCATTERING DETAIL AROUND THE
6380  REM   NORMALIZED RANGE.   PLOT SELECTION IS GOVERNED BY A REPETITION
6390  REM   INDEX, Rpt, AND IS CONTROLLED IN THE CALLING MODULE (OUTPT).
6400  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
6410  !
6420  !++++ MODULE VARIABLE DECLARATIONS AND DEFINITIONS +++++++++++++++++++++
6430                                      !
6440                                      !
6450  REAL Xgumax                         ! THE MAXIMUM ABSCISSA VALUE IN
6460                                      ! GRAPHICS DISPLAY UNITS.
6470                                      !
6480  REAL Ygumax                         ! THE MAXIMUM ORDINATE VALUE IN
6490                                      ! GRAPHICS DISPLAY UNITS.
6500                                      !
6510  REAL Gumax                          ! THE MINIMUM OF XGUMAX AND YGUMAX;
6520                                      ! USED TO DETERMINE PLOTTING AREA.
6530                                      !
6540  REAL Dtmx                           ! THE MAX. VALUE OF THE ARRAY DS(*).
6550                                      !
6560  REAL Endpt                          ! THE MAX. RADIAL SCALE VALUE IN
6570                                      ! USER DEFINED UNITS.
6580                                      !
6590  REAL Ring                           ! GRID RING SPACING IN USER UNITS.
6600                                      !
6610  REAL Tick                           ! RADIAL AXIS TICK MARK SPACINGS IN
6620                                      ! USER DEFINED UNITS.
6630                                      !
6640                                      !
6650  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
6660  !
6670  !++++ INITIALIZING THE PLOTTER +++++++++++++++++++++++++++++++++++++++++
6680                                      !
6690  GINIT
6700  PLOTTER IS CRT,"INTERNAL"
6710  GRAPHICS ON
6720  GCLEAR
6730                                      !
6740  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
6750  !
6760  !++++ GRAPH CONSTRUCTION AND LABELLING ++++++++++++++++++++++++++++++++++
6770                                        !
6780                                        !
6790  Xgumax=100*MAX(1,RATIO)
6800  Ygumax=100*MAX(1,1/RATIO)
6810  Gumax=MIN(Xgumax,Ygumax)
6820                                        !
6830  CSIZE 3                               ! ** LABELLING MAJOR RADIALS **
6840  LINE TYPE 1
6850  MOVE Gumax,Gumax/2
6860  LORG 8
6870  LABEL "0"
6880  MOVE Gumax/2,Gumax
6890  LORG 6
6900  LABEL "PI/2"
6910  MOVE 0,Gumax/2
6920  LORG 2
6930  LABEL "PI"
6940  MOVE Gumax/2,0
6950  LORG 4
6960  LABEL "3*PI/2"
6970                                        ! ** DEFINES THE TOTAL PLOTTING **
6980  VIEWPORT 5,Gumax-5,5,Gumax-5          ! ** AREA IN GRAPH UNITS.       **
6990                                        !
7000                                        ! ** SCALING THE GRAPH TO **
7010                                        ! ** USER UNITS.          **
7020  Dtmax=0                               ! DETERMINING MAX. RADIUS VALUE
7030  IF Rpt=1 THEN                         ! REQ'D FOR THE PLOT.
7040    FOR I=0 TO 360
7050      IF Dtmx<Ds(I) THEN Dtmx=Ds(I)
7060    NEXT I
7070  END IF
7080  Endpt=INT(Dtmx+1)
7090  IF Endpt<2 THEN Endpt=2
7100  SHOW -Endpt,Endpt,-Endpt,Endpt        ! ISOTROPICALLY SCALES THE GRAPH.
7110                                        !
7120  LINE TYPE 4                           ! ** MAJOR RADIALS CONSTRUCTION **
7130  Ring=INT(Endpt/5+1)
7140  Tick=.2*Ring
7150  AXES Tick,Tick,0,0,5,5,2
7160                                        !
7170  LINE TYPE 3                           ! ** RANGE RING CONSTRUCTION **
7180  DEG
7190  LORG 7
7200  FOR R=1 TO Endpt/Ring
7210    LINE TYPE 1
7220    MOVE 0,R*Ring
7230    LABEL R*Ring                        ! LABELLING THE RANGE.
7240    LINE TYPE 3
7250    FOR Angle=0 TO 360                  ! DRAWING THE RING.
7260      PLOT R*Ring*COS(Angle),R*Ring*SIN(Angle)
7270    NEXT Angle
7280    PENUP
7290  NEXT R
```

```
7300                                        !
7310  FOR Angle=30 TO 150 STEP 30           ! ** 30 DEGREE RADIALS **
7320    IF Angle<>90 THEN                   ! ** CONSTRUCTION.     **
7330      PLOT Endpt*COS(Angle),Endpt*SIN(Angle),-2
7340      DRAW Endpt*COS(180+Angle),Endpt*SIN(180+Angle)
7350    END IF
7360  NEXT Angle
7370  PENUP
7380                                        !
7390                                        !
7400  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7410  !
7420  !++++ PLOTTING THE CALCULATED DSCS +++++++++++++++++++++++++++++++++++++
7430                                        !
7440  LINE TYPE 1
7450  DEG
7460  FOR I=0 TO 360                        ! PLOTTING RECTANGULAR COORDINATES
7470    PLOT Ds(I)*COS(I),Ds(I)*SIN(I)      ! IN POLAR FORMAT.
7480  NEXT I
7490                                        !
7500  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7510  !
7520  !++++ PRINTING THE PLOT ++++++++++++++++++++++++++++++++++++++++++++++++
7530                                        !
7540  DUMP GRAPHICS #701
7550                                        !
7560  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7570  !
7580  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
7590                                        !
7600  GRAPHICS OFF
7610  SUBEND
7620                                        !
7630  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7640  !
7650  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
7660  !
```

```
7670  !
7680  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
7690  SUB Recplt(Sm(*))
7700  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7710  REM   THIS MODULE MAPS THE CALCULATED SCATTERING MODULUS ARRAY SM(*) ON
7720  REM   A SEMI-LOG PLOT.
7730  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7740  !
7750  !++++ MODULE VARIABLE DECLARATIONS AND DEFINITIONS ++++++++++++++++++++
7760                                      !
7770                                      !
7780  REAL Xgumax                         ! THE MAXIMUM ABSCISSA VALUE IN
7790                                      ! GRAPHICS DISPLAY UNITS.
7800                                      !
7810  REAL Ygumax                         ! THE MAXIMUM ORDINATE VALUE IN
7820                                      ! GRAPHICS DISPLAY UNITS.
7830                                      !
7840  REAL Dtmx                           ! THE MAX. VALUE OF THE ARRAY SM(*).
7850                                      !
7860  REAL Dtmn                           ! THE MIN. VALUE OF THE ARRAY SM(*).
7870                                      !
7880  REAL Mxy                            ! THE MAX. ORDINATE SCALE VALUE IN
7890                                      ! USER DEFINED UNITS.
7900                                      !
7910  REAL Mny                            ! THE MIN. ORDINATE SCALE VALUE IN
7920                                      ! USER DEFINED UNITS.
7930                                      !
7940                                      !
7950  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7960  !
7970  !++++ INITIALIZING THE PLOTTER ++++++++++++++++++++++++++++++++++++++++
7980                                      !
7990  GINIT
8000  PLOTTER IS CRT,"INTERNAL"
8010  GRAPHICS ON
8020  GCLEAR
8030                                      !
8040  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8050  !
8060  !++++ GRAPH CONSTRUCTION AND LABELLING ++++++++++++++++++++++++++++++++
8070                                      !
8080                                      !
8090  Xgumax=100*MAX(1,RATIO)
8100  Ygumax=100*MAX(1,1/RATIO)
8110                                      !
8120  CSIZE 4
8130                                      !
8140  LORG 4                              ! ** LABELLING THE ABSCISSA AXIS **
8150  MOVE Xgumax/2,0
8160  LABEL "PHI (DEGREES)"
```

```
8170                                        !
8180  DEG                                   ! ** LABELLING THE ORDINATE AXIS **
8190  LDIR 90
8200  LORG 6
8210  MOVE 0,Ygumax/2
8220  LABEL "SCTRG MODULUS"
8230  LDIR 0
8240                                        ! ** GRAPH CONSTRUCTION **
8250  VIEWPORT 15,Xgumax-5,10,Ygumax-5      ! DEFINES THE TOTAL PLOTTING AREA
8260                                        ! IN GRAPH UNITS.
8270                                        !
8280  Dtmx=0                                ! ** ORDINATE USER UNITS **
8290  Dtmn=Sm(180)                          ! ** DETERMINED.         **
8300  FOR I=0 TO 180
8310    IF Dtmx<Sm(I) THEN Dtmx=Sm(I)
8320    IF Dtmn>Sm(I) THEN Dtmn=Sm(I)
8330  NEXT I
8340  Mxy=INT(LGT(Dtmx))+1
8350  Mny=INT(LGT(Dtmn))
8360                                        !
8370  WINDOW 0,180,Mny,Mxy                  ! ANISOTROPICALLY SCALES THE GRAPH
8380                                        ! TO USER UNITS.
8390  AXES 30,1,0,Mny,1,1,1.5               ! PLOTS THE AXES.
8400                                        !
8410  LINE TYPE 3
8420  FOR I=Mny TO Mxy                      ! * PLOTS THE HORIZONTAL          *
8430    FOR J=1 TO 9                        ! * LOGARITHMIC GRID LINES I.A.W. *
8440      MOVE 0,I+LGT(J)                   ! * THE VERTICAL SCALE.           *
8450      PLOT 180,I+LGT(J),-1
8460      PENUP
8470    NEXT J
8480  NEXT I
8490                                        !
8500  FOR I=60 TO 180 STEP 60               ! * PLOTS THE VERTICAL GRID LINES *
8510    MOVE I,Mny                          ! * I.A.W. THE HORIZONTAL SCALE.  *
8520    PLOT I,Mxy,-1
8530    PENUP
8540  NEXT I
8550                                        !
8560  CLIP OFF
8570  LINE TYPE 1
8580                                        !
8590  LORG 6                                ! ** LABELLING THE ABSCISSA SCALE **
8600  FOR I=0 TO 180 STEP 30
8610    MOVE I,Mny
8620    LABEL I
8630  NEXT I
8640                                        !
8650  LORG 8                                ! ** LABELLING THE ORDINATE SCALE **
8660  FOR I=Mny TO Mxy
8670    MOVE 0,I
8680    LABEL 10^I                          ! POWER OF 10 LABELS.
8690    MOVE 0,I+LGT(5)
8700    LABEL (10^(I+1))/2                  ! HALF THE NEXT POWER OF 10 LABELS.
8710  NEXT I
```

```
8720                                      !
8730  CLIP ON
8740                                      !
8750                                      !
8760  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8770  !
8780  !++++ PLOTTING THE CALCULATED SCATTERING MODULUS +++++++++++++++++++++++
8790                                      !
8800  LINE TYPE 1                         ! THIS PLOT IS FOR THE PLANE WAVE
8810  FOR I=0 TO 180                      ! INCIDENT FROM THE RIGHT (0 OEG'S).
8820    PLOT I,LGT(Sm(I+180))             ! USE SM(I) IN PLACE OF SM(I+180)
8830  NEXT I                              ! FOR A PLANE WAVE INCIOENT FROM THE
8840                                      ! LEFT (180 OEG'S).
8850                                      !
8860  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8870  !
8880  !++++ PRINTING THE PLOT ++++++++++++++++++++++++++++++++++++++++++++++++
8890                                      !
8900  OUMP GRAPHICS #701
8910                                      !
8920  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8930  !
8940  !++++ MOOULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
8950                                      !
8960  GRAPHICS OFF
8970  SUBEND
8980                                      !
8990  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
9000  !
9010  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
```

APPENDIX C

BACKSCATTERED CROSS SECTION PROGRAM

Examples of the output of this program are Figures 6 and 7, located in Chapter III.

```
10      !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
20      !
30      REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
40      REM     ACOUSTICALLY HARD SPHERE BACKSCATTERING OF PLANE ACOUSTIC WAVES
50      REM
60      REM     THIS PROGRAM CALCULATES AND GRAPHS THE NORMALIZED, BACKSCATTERED
70      REM   CROSS SECTION FOR ACOUSTICALLY HARD SPHERES OF A SELECTED RANGE
80      REM   OF DIAMETERS D ENSONIFIED BY AN INCIDENT PLANE WAVE.
90      REM
100     REM
110     REM
120     REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
130     !
140     !++++ MAIN PROGRAM VARIABLE DECLARATION AND DEFINITION +++++++++++++++++
150                                         !
160                                         !
170     REAL Nbs(301)                       ! THE ARRAY REPRESENTING THE
180                                         ! NORMALIZED, BACKSCATTERED CROSS
190                                         ! SECTION (NBCS).
200                                         !
210     REAL K                              ! THE WAVE NUMBER, 2*PI*FREQ/C.
220                                         !
230     REAL A                              ! RADIUS OF THE SPHERE.
240                                         !
250     REAL Ka                             ! K*A, THE ARGUMENT OF THE REGULAR
260                                         ! AND IRREGULAR SPHERICAL BESSEL
270                                         ! FUNCTIONS AND THEIR DERIVATIVES.
280                                         !
290     REAL Kamin                          ! INTEGER OF THE MIN. VALUE OF KA.
300                                         !
310     REAL Kamax                          ! INTEGER OF THE MAX. VALUE OF KA.
320                                         !
330     REAL Stp                            ! KA INCREMENT.
340                                         !
350                                         !
360     !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
370     !
```

```
380   !++++ MAIN PROGRAM +++++++++++++++++++++++++++++++++++++++++++++++++++++
390                                       !
400   CALL Bcksctr(Nbs(*),Kamin,Kamax,Stp)! CALCULATE THE NBS(*).
410                                       !
420   CALL Recplt(Nbs(*),Kamin,Kamax,Stp) ! OUTPUT THE NBS(*).
430                                       !
440   !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
450   !
460   !++++ MAIN PROGRAM CONCLUSION ++++++++++++++++++++++++++++++++++++++++++
470                                       !
480   PRINTER IS 701
490   PRINT CHR$(12)
500   END
510                                       !
520   !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
530   !
540   !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
550   !
```

```
560   !
570   !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
580   SUB Bcksctr(Nbs(*),Kamin,Kamax,Stp)
590   REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
600   REM   THIS MODULE CALCULATES THE ARRAY NBS(*) REPRESENTING THE
610   REM   BACKSCATTERED CROSS SECTION NORMALIZED BY THE CROSS SECTIONAL AREA
620   REM   OF THE SPHERE.
630   REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
640   !
650   !++++ MODULE VARIABLE DECLARATIONS AND DEFINITIONS ++++++++++++++++++++
660                                     !
670                                     !
680   REAL L                            ! ORDER OF THE FUNCTION CALCULATED
690                                     ! IN A SUBROUTINE.  IT IS USED
700                                     ! AS A LOOP INDEX AND TO DESIGNATE
710                                     ! THE ARRAY ELEMENT FOR THE FUNCTION
720                                     ! OF CORRESPONDING ORDER.
730                                     !
740   REAL Lmax                         ! MAX. SIGNIFICANT ORDER OF THE SUM
750                                     ! DETERMINING THE DIFFERENTIAL
760                                     ! SCATTERING CROSS SECTION (DSCS)
770                                     ! AND SCATTERING MODULUS (SM).
780                                     !
790   REAL J(51)                        ! ARRAY FOR THE REGULAR SPHERICAL
800                                     ! BESSEL FUNCTIONS (RSB) OF
810                                     ! ARGUMENT KA AND ORDER 0 THROUGH
820                                     ! LMAX.
830                                     !
840   REAL Dj(51)                       ! ARRAY FOR THE DERIVATIVES OF THE
850                                     ! RSB (THE DRSB).
860                                     !
870   REAL Y(51)                        ! ARRAY FOR THE IRREGULAR SPHERICAL
880                                     ! BESSEL FUNCTIONS (ISB) OF
890                                     ! ARGUMENT KA AND ORDER 0 THROUGH
900                                     ! LMAX.
910                                     !
920   REAL Dy(51)                       ! ARRAY FOR THE DERIVATIVES OF THE
930                                     ! ISB (THE DISB).
940                                     !
950   REAL P(51)                        ! ARRAY FOR THE LEGENDRE
960                                     ! POLYNOMIALS OF ARGUMENT -1.
970                                     !
980   REAL Rsm                          ! THE REAL COMPONENT OF THE
990                                     ! SCATTERING MODULUS.
1000                                    !
1010  REAL Ism                          ! THE IMAGINARY COMPONENT OF THE
1020                                    ! SCATTERING MODULUS.
1030                                    !
1040  REAL Fac                          ! FACTOR COMMON TO RSM AND ISM.
1050                                    !
1060                                    ! ++++ NOTE ++++
1070                                    ! THE ARRAY DIMENSIONS (30+21) NEED
1080                                    ! TO BE INCREASED TO ACCOMODATE
1090                                    ! KAMAX>30.
```

```
1100                                    !
1110                                    !
1120  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1130  !
1140  !++++ CALCULATING THE NBCS ARRAY +++++++++++++++++++++++++++++++++++++++
1150                                    !
1160                                    !
1170  PRINTER IS 1
1180  INPUT Kamin,Kamax                 ! DETERMINE DESIRED RANGE OF KA
1190  Kamin=INT(Kamin)                  ! VALUES.
1200  Kamax=INT(Kamax)
1210  Stp=.1
1220                                    !
1230  FOR Ka=Kamin+Stp TO Kamax STEP Stp
1240    Lmax=INT(Ka+21)
1250    CALL Drsb(Lmax,Ka,Dj(*),J(*))   ! CALCULATE THE DRSB OF ARGUMENT KA
1260                                    ! AND ORDERS 0 THROUGH LMAX.
1270    CALL Disb(Lmax,Ka,Dy(*),Y(*))   ! CALCULATE THE DISB OF ARGUMENT KA
1280                                    ! AND ORDERS 0 THROUGH LMAX.
1290    CALL Leg180(Lmax,Ka,P(*))       ! CALCULATE P(*) OF ARGUMENT -1 AND
1300                                    ! ORDERS 0 THROUGH LMAX.
1310    Rsm=0                           ! CALCULATING THE REAL AND IMAGINARY
1320    Ism=0                           ! COMPONENTS OF THE SCATTERING
1330    FOR L=0 TO Lmax                 ! MODULUS.
1340      Fac=(Dj(L)*(2*L+1)*P(L))/(Dj(L)*Dj(L)+Dy(L)*Dy(L))
1350      Rsm=Rsm+(Fac*Dy(L))
1360      Ism=Ism+(Fac*Dj(L))
1370    NEXT L
1380                                    ! CALCULATING THE ARRAY REPRESENTING
1390                                    ! THE NBCS.
1400    Nbs((Ka-Kamin)/Stp)=4*(Rsm*Rsm+Ism*Ism)/(Ka*Ka)
1410                                    !
1420  NEXT Ka
1430                                    !
1440                                    !
1450  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1460  !
1470  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
1480                                    !
1490  SUBEND
1500                                    !
1510  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1520  !
1530  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
1540  !
```

```
1550  !
1560  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
1570  SUB Drsb(Lmax,Ka,Dj(*),J(*))
1580  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1590  REM   THIS MODULE CALCULATES THE DERIVATIVES OF THE REGULAR SPHERICAL
1600  REM   BESSEL FUNCTIONS (DRSB) OF ARGUMENT KA AND ORDERS 0 THROUGH LMAX
1610  REM   THE CALCULATION OF THE DERIVATIVE INVOLVES THE RSB'S OF THE SAME
1620  REM   AND PREVIOUS ORDER.
1630  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1640  !
1650  !++++ CALCULATING THE DERIVATIVES OF THE RSB ++++++++++++++++++++++++++++
1660                                      !
1670                                      !
1680  CALL Rsb(Lmax,Ka,J(*))              ! CALCULATE THE RSB VALUES J(*).
1690                                      !
1700  RAD                                 ! ANGLE UNITS FOR DRSB CALC'NS.
1710  Dj(0)=(-J(0)+COS(Ka))/Ka            ! CALCULATE THE INITIAL DRSB DJ(0).
1720  FOR L=1 TO Lmax                     ! CALCULATE THE REMAINING DRSB.
1730    Dj(L)=-((L+1)/Ka)*J(L)+J(L-1)
1740  NEXT L
1750                                      !
1760                                      !
1770  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1780  !
1790  !++++ MODULE CONCLUSION +++++++++++++++++++++++++++++++++++++++++++++++++
1800                                      !
1810  SUBEND
1820                                      !
1830  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1840  !
1850  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
1860  !
```

```
1870  !
1880  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
1890  SUB Rsb(Lmax,Ka,J(*))
1900  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1910  REM   THIS MODULE CALCULATES THE REGULAR SPHERICAL BESSEL FUNCTIONS
1920  REM   (RSB) OF ARGUMENT KA AND ORDERS 0 THROUGH LMAX.   THE
1930  REM   CALCULATION USES THE CONTINUED FRACTION APPROACH.
1940  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1950  !
1960  !++++ MODULE VARIABLE DECLARATIONS AND DEFINITIONS +++++++++++++++++++++
1970                                        !
1980                                        !
1990  REAL Rl                               ! THE EVOLVING RATIO OF J(0)/J(L).
2000                                        !
2010  REAL Numr                             ! THE EVOLVING NUMERATOR FACTOR USED
2020                                        ! IN CALCULATION OF RL.
2030                                        !
2040  REAL Denr                             ! THE EVOLVING DENOMINATOR FACTOR
2050                                        ! USED IN CALCULATION OF RL.
2060                                        !
2070  REAL Scexp                            ! EVOLVING SCALING EXPONENT FOR VERY
2080                                        ! LARGE OR VERY SMALL VALUES OF RL.
2090                                        !
2100  REAL Nflag                            ! 0 FLAG FOR NUMR.
2110                                        !
2120  REAL Dflag                            ! 0 FLAG FOR DENR.
2130                                        !
2140  REAL Bnm                              ! EVOLVING TERM USED IN THE
2150                                        ! CALCULATION OF NUMR AND DENR.
2160                                        !
2170  REAL Binc                             ! INCREMENT USED IN THE EVOLUTION
2180                                        ! OF BNM.
2190                                        !
2200                                        !
2210  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
2220  !
2230  !++++ CALCULATING THE RSB ++++++++++++++++++++++++++++++++++++++++++++++
2240                                        !
2250                                        !
2260  RAD                                   ! ANGLE UNITS FOR CALC'NS.
2270                                        !
2280                                        ! *** CALCULATE THE INITIAL ***
2290  J(0)=(SIN(Ka))/Ka                     ! *** RSB, J(0).            ***
2300                                        !
2310                                        ! *** CALCULATING THE RATIO ***
2320  FOR L=1 TO Lmax                       ! *** J(0)/J(L).            ***
2330    Rl=1.0                              ! INITIALIZING RL.
2340    Numr=0.                             ! INITIALIZING NUMR.
2350    Denr=0.                             ! INITIALIZING DENR.
2360    Scexp=0.                            ! INITIALIZING SCEXP.
2370    Nflag=0.                            ! INITIALIZING NFLAG.
2380    Dflag=0.                            ! INITIALIZING DFLAG.
2390    Bnm=1.0/Ka                          ! INITIALIZING BNM.
2400    Binc=2.0/Ka                         ! CALCULATE BINC.
2410                                        !
```

```
2420                                         ! * CALCULATE THE L "NAKED"    *
2430    FOR Ordcnt=1 TO L                    ! * NUMR FACTORS OF J(0)/J(L). *
2440      Bnm=Bnm+Binc                       ! INCREMENT BNM.
2450      IF Nflag=0 THEN                    ! CHECK NUMR=0 FLAG NOT SET;
2460        Numr=Bnm-Numr                    ! UPDATE NUMR IF NUMR<>0.
2470        IF Numr<>0 THEN
2480          Rl=Rl*Numr                     ! UPDATE RL IF UPDATED NUMR<>0.
2490          Numr=1.0/Numr                  ! PREPARE NUMR FOR NEXT EVOLUTION.
2500                                         ! CHECK SCALING OF UPDATED RL.
2510          IF ABS(Rl)>10^250 THEN CALL Scale(Rl,Scexp)
2520        ELSE
2530          Nflag=1
2540        END IF
2550      ELSE                               ! IF NFLAG<>0 THE PREVIOUS STEPS ARE
2560        Nflag=0                          ! DEFERRED FOR ONE INCREMENT OF BNM.
2570      END IF
2580    NEXT Ordcnt
2590                                         ! * CALCULATE THE REMAINING NUMR *
2600                                         ! * AND DENR TERMS OF J(0)/J(L). *
2610                                         !   NOTE WHEN NUMR=DENR THEN
2620                                         !   RL=J(0)/J(L).
2630    WHILE Numr<>Denr OR Nflag=1 OR Dflag=1
2640      Bnm=Bnm+Binc                       ! INCREMENT BNM.
2650                                         !
2660      IF Nflag=0 THEN                    ! CHECK NUMR=0 FLAG NOT SET;
2670        Numr=Bnm-Numr                    ! UPDATE NUMR IF NUMR<>0.
2680        IF Numr<>0 THEN
2690          Rl=Rl*Numr                     ! UPDATE RL IF UPDATED NUMR<>0.
2700          Numr=1.0/Numr                  ! PREPARE NUMR FOR NEXT EVOLUTION.
2710                                         ! CHECK SCALING OF UPDATED RL.
2720          IF ABS(Rl)>10^250 THEN CALL Scale(Rl,Scexp)
2730        ELSE
2740          Nflag=1
2750        END IF
2760      ELSE                               ! IF NFLAG<>0 THE PREVIOUS STEPS ARE
2770        Nflag=0                          ! DEFERRED FOR ONE INCREMENT OF BNM.
2780      END IF
2790                                         !
2800      IF Dflag=0 THEN                    ! CHECK DENR=0 FLAG NOT SET;
2810        Denr=Bnm-Denr                    ! UPDATE DENR IF DENR<>0.
2820        IF Denr<>0 THEN
2830          Rl=Rl/Denr                     ! UPDATE RL IF UPDATED DENR<>0.
2840          Denr=1.0/Denr                  ! PREPARE DENR FOR NEXT EVOLUTION.
2850                                         ! CHECK SCALING OF UPDATED RL.
2860          IF ABS(Rl)>10^250 THEN CALL Scale(Rl,Scexp)
2870        ELSE
2880          Dflag=1
2890        END IF
2900      ELSE                               ! IF DFLAG<>0 THE PREVIOUS STEPS ARE
2910        Dflag=0                          ! DEFERRED FOR ONE INCREMENT OF BNM.
2920      END IF
2930                                         !
2940    END WHILE
```

```
2950                                      !
2960                                      !
2970    J(L)=J(0)/(R1*10^(Scexp))         ! *** J(L) DETERMINED. ***
2980                                      !
2990  NEXT L
3000                                      !
3010                                      !
3020  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3030  !
3040  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
3050                                      !
3060  SUBEND
3070                                      !
3080  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3090  !
3100  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
3110  !
```

```
3120  !
3130  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
3140  SUB Scale(R1,Scexp)
3150  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3160  REM   THIS MODULE SCALES RL TO MAINTAIN NUMERICAL ACCURACY IN THE
3170  REM   ONGOING CALCULATION OF RL.
3180  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3190  !
3200  !++++ SCALING RL +++++++++++++++++++++++++++++++++++++++++++++++++++++++
3210                                      !
3220                                      !
3230  IF R1>10^250 THEN
3240    R1=R1*10^(-250)
3250    Scexp=Scexp+250
3260  END IF
3270  IF R1<10^(-250) THEN
3280    R1=R1*10^250
3290    Scexp=Scexp-250
3300  END IF
3310                                      !
3320                                      !
3330  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3340  !
3350  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
3360                                      !
3370  SUBEND
3380                                      !
3390  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3400  !
3410  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
3420  !
```

```
3430  !
3440  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
3450  SUB Disb(Lmax,Ka,Dy(*),Y(*))
3460  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3470  REM   THIS MODULE CALCULATES THE DERIVATIVES OF THE IRREGULAR SPHERICAL
3480  REM   BESSEL FUNCTIONS OF ARGUMENT KA AND ORDERS 0 THROUGH LMAX.  THE
3490  REM   CALCULATION OF THE DERIVATIVE USES THE ISB'S OF THE SAME AND
3500  REM   PREVIOUS ORDER.
3510  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3520  !
3530  !++++ CALCULATING THE DERIVATIVES OF THE ISB +++++++++++++++++++++++++++
3540                                       !
3550                                       !
3560  CALL Isb(Lmax,Ka,Y(*))               ! CALCULATE THE ISB VALUES.
3570                                       !
3580  RAD                                  ! ANGLE UNITS FOR CALC'NS.
3590  Dy(0)=(-Y(0)+SIN(Ka))/Ka             ! THE INITIAL ISB DERIVATIVE.
3600  FOR L=1 TO Lmax                      ! THE REMAINING ISB DERIVATIVES.
3610    Dy(L)=-((L+1)/Ka)*Y(L)+Y(L-1)
3620  NEXT L
3630                                       !
3640                                       !
3650  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3660  !
3670  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
3680                                       !
3690  SUBEND
3700                                       !
3710  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3720  !
3730  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
3740  !
```

```
3750  !
3760  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
3770  SUB Isb(Lmax,Ka,Y(*))
3780  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3790  REM   THIS MODULE CALCULATES THE IRREGULAR SPHERICAL BESSEL FUNCTIONS
3800  REM   OF ARGUMENT KA AND  ORDER L THROUGH LMAX.   THE CALCULATION IS
3810  REM   ACCOMPLISHED BY FORWARD RECURSION.
3820  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3830  !
3840  !++++ CALCULATING THE ISB ++++++++++++++++++++++++++++++++++++++++++++++
3850                                      !
3860                                      !
3870  RAD                                 ! ANGLE UNITS FOR CALC'NS.
3880  Y(0)=-COS(Ka)/Ka                    ! INITIAL ISB'S CALCULATED.
3890  Y(1)=(Y(0)-SIN(Ka))/Ka
3900  FOR L=2 TO Lmax                     ! REMAINING ISB'S CALC'D.
3910    Y(L)=((2*L-1)/Ka)*Y(L-1)-Y(L-2)
3920  NEXT L
3930                                      !
3940                                      !
3950  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3960  !
3970  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
3980                                      !
3990  SUBEND
4000                                      !
4010  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4020  !
4030  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
4040  !
```

```
4050  !
4060  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
4070  SUB Leg180(Lmax,Ka,P(*))
4080  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4090  REM   THIS MODULE CALCULATES THE LEGENDRE POLYNOMIALS OF ARGUMENT -1 AND
4100  REM   ORDER 0 THROUGH LMAX.  CALCULATION IS ACCOMPLISHED BY FORWARD
4110  REM   RECURSION.
4120  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4130  !
4140  !++++ CALCULATING THE LEGENDRE POLYNOMIALS +++++++++++++++++++++++++++++
4150                                      !
4160  X=-1                                ! ARGUMENT OF THE LEG. POL'S.
4170                                      !
4180  P(0)=1                              ! INITIAL LEG. POL'S. CALC'D.
4190  P(1)=X
4200  FOR L=2 TO Lmax                     ! REMAINING LEG. POL'S. CALC'D.
4210    P(L)=(2-(1/L))*X*P(L-1)-(1-(1/L))*P(L-2)
4220  NEXT L
4230                                      !
4240  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4250  !
4260  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
4270                                      !
4280  SUBEND
4290                                      !
4300  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4310  !
4320  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
4330  !
```

```
4340  !
4350  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
4360  SUB Recplt(Nbs(*),Kamin,Kamax,Stp)
4370  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4380  REM   THIS MODULE MAPS THE CALCULATED SCATTERING MODULUS ARRAY SM(*) ON
4390  REM   A SEMI-LOG PLOT.
4400  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4410  !
4420  !++++ MODULE VARIABLE DECLARATIONS AND DEFINITIONS ++++++++++++++++++++++
4430                                       !
4440                                       !
4450  REAL Xgumax                          ! THE MAXIMUM ABSCISSA VALUE IN
4460                                       ! GRAPHICS DISPLAY UNITS.
4470                                       !
4480  REAL Ygumax                          ! THE MAXIMUM ORDINATE VALUE IN
4490                                       ! GRAPHICS DISPLAY UNITS.
4500                                       !
4510                                       !
4520  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4530  !
4540  !++++ INITIALIZING THE PLOTTER ++++++++++++++++++++++++++++++++++++++++++
4550                                       !
4560  GINIT
4570  PLOTTER IS CRT,"INTERNAL"
4580  GRAPHICS ON
4590  GCLEAR
4600                                       !
4610  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4620  !
4630  !++++ GRAPH CONSTRUCTION AND LABELLING ++++++++++++++++++++++++++++++++++
4640                                       !
4650                                       !
4660  Xgumax=100*MAX(1,RATIO)
4670  Ygumax=100*MAX(1,1/RATIO)
4680                                       !
4690  CSIZE 4
4700  LINE TYPE 1
4710                                       !
4720  LORG 4                               ! ** LABELLING THE ABSCISSA AXIS **
4730  MOVE Xgumax/2,0
4740  LABEL "K*A"
4750                                       !
4760  DEG                                  ! ** LABELLING THE ORDINATE AXIS **
4770  LDIR 90
4780  LORG 6
4790  MOVE 0,Ygumax/2
4800  LABEL "NORM'D BCKSCTRG CROSS SEC'N"
4810  LDIR 0
4820                                       ! ** GRAPH CONSTRUCTION **
4830  VIEWPORT 10,Xgumax-5,10,Ygumax-5     ! DEFINES THE TOTAL PLOTTING AREA
4840                                       ! IN GRAPH UNITS.
4850                                       !
4860  WINDOW Kamin,Kamax,0,1.2
4870  LINE TYPE 4
4880  GRID Stp*10,.1,Kamin,0,61,10,.2
```

```
4890                                      !
4900  CLIP OFF
4910  CSIZE 3
4920  LINE TYPE 1
4930                                      !
4940  LORG 6                              ! ** LABELLING THE ABSCISSA SCALE **
4950  FOR I=Kamin TO Kamax STEP Stp*20
4960    MOVE I ,0
4970    LABEL I
4980  NEXT I
4990                                      !
5000  LORG 8                              ! ** LABELLING THE ORDINATE SCALE **
5010  FOR I=0 TO 1 STEP .5
5020    MOVE Kamin,I
5030    LABEL I
5040  NEXT I
5050                                      !
5060  CLIP ON
5070                                      !
5080                                      !
5090  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5100  !
5110  !++++ PLOTTING THE CALCULATED NBCS +++++++++++++++++++++++++++++++++++++
5120                                      !
5130  FOR I=Kamin+Stp TO Kamax STEP Stp
5140    PLOT I ,Nbs((I-Kamin)/Stp)
5150  NEXT I
5160                                      !
5170  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5180  !
5190  !++++ PRINTING THE PLOT ++++++++++++++++++++++++++++++++++++++++++++++++
5200                                      !
5210  DUMP GRAPHICS #701
5220                                      !
5230  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5240  !
5250  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
5260                                      !
5270  GRAPHICS OFF
5280  SUBEND
5290                                      !
5300  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5310  !
5320  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
```

APPENDIX D

ACOUSTIC ARRAY EFFICIENCY PROGRAM

This program was used to analyze data and determine the calibration product $E_r E_t$. The output of this program was also used to construct Figures 13, 14, and 15.

The power supply mentioned in the Gain30 and Gain35 subprograms was the HP3314A function generator. All data contained in the data subprograms are repeated in Appendix E. Data units are available in Appendix E with their respective data.

```
10    !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
20    !
30    REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
40    REM                    ACOUSTIC ARRAY EFFICIENCY
50    REM
60    REM      THIS PROGRAM CALCULATES THE EFFICIENCY AND CENTERLINE GAIN OF
70    REM   AN ACOUSTIC ARRAY BASED ON THE RETURN FROM AN ACOUSTICALLY HARD
80    REM   SPHERE OF DIAMETER D ENSONIFIED BY AN INCIDENT PLANE WAVE OF
90    REM   FREQUENCY F.
100   REM
130   REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
140   !
150   !++++ VARIABLE DECLARATIONS AND DEFINITIONS ++++++++++++++++++++++++++++
160                                       !
170                                       !
180   REAL Vmsr                           ! MEAN SQUARE VOLTAGE RECEIVED FROM
190                                       ! THE TARGET SPHERE AND MEASURED
200                                       ! AFTER PRE-AMPLIFICATION.
210                                       !
220   REAL Vmsn                           ! MEAN SQUARE VOLTAGE RECEIVED IN
230                                       ! THE ABSCENCE OF A TARGET AND
240                                       ! MEASURED AFTER PRE-AMP.
250                                       !
```

```
260  REAL Vmst                                ! MEAN SQUARE VOLTAGE TRANSMITTED.
270                                           !
280  REAL R                                   ! THE RANGE FROM THE PLANE OF THE
290                                           ! ARRAY SPEAKER DIAPHRAGMS TO THE
300                                           ! CENTER OF THE TARGET SPHERE AND TO
310                                           ! THE SPECTRUM ANALYZER MICROPHONE.
320                                           !
330  REAL Et                                  ! THE EFFICIENCY OF CONVERSION
340                                           ! FROM TRANSMITTED ELECTRICAL POWER
350                                           ! TO TRANSMITTED ACOUSTICAL POWER.
360                                           !
370  REAL Go                                  ! THE CENTERLINE GAIN OF THE ACOUSTIC
380                                           ! ARRAY.
390                                           !
400  REAL Etgo                                ! ET*GO
410                                           !
420  REAL At                                  ! THE ATTENUATION COEFFICIENT,
430                                           ! "ALPHA".
440                                           !
450  REAL C,Cmoist                            ! THE SPEED OF SOUND IN HUMID AIR.
460                                           !
470  REAL Efreq                               ! THE SELECTED FREQUENCY OF THE
480                                           ! ACOUSTIC ARRAY.
490                                           !
500  REAL K                                   ! THE WAVE NUMBER, 2*PI*EFREQ/C.
510                                           !
520  REAL A                                   ! THE RADIUS OF THE TARGET SPHERE.
530                                           !
540  REAL Ka                                  ! K*A, THE ARGUMENT OF THE REGULAR
550                                           ! AND IRREGULAR SPHERICAL BESSEL
560                                           ! FUNCTIONS AND THEIR DERIVATIVES.
570                                           !
580  REAL Aa                                  ! THE APERATURE AREA OF THE ARRAY.
590                                           !
600  REAL Gpre                                ! THE PRE-AMP'S ELECTRICAL GAIN.
610                                           !
620  REAL Nbs                                 ! THE VARIABLE REPRESENTING THE
630                                           ! NORMALIZED, BACKSCATTERED CROSS
640                                           ! SECTION (NBCS).
650                                           !
660  REAL Atgt                                ! CROSS SECTIONAL AREA OF THE
670                                           ! TARGET SPHERE.
680                                           !
690  REAL Sasph                               ! SURFACE AREA OF AN IMAGINARY
700                                           ! SPHERE CENTERED ON THE ARRAY
710                                           ! AND HAVING A RADIUS EQUAL TO
720                                           ! THE RANGE TO THE CENTER OF THE
730                                           ! TARGET SPHERE.
740                                           !
750  REAL Prat                                ! THE RATIO OF RECEIVED TARGET
760                                           ! ELECTRICAL POWER TO THE
770                                           ! TRANSMITTED ELECTRICAL POWER.
780                                           !
```

```
790    REAL Er                              ! THE EFFICIENCY OF CONVERSION
800                                         ! FROM RECEIVED ACOUSTICAL POWER
810                                         ! TO RECEIVED ELECTRICAL POWER.
820                                         !
830                                         !
840    !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
850    !
860    !++++ MAIN PROGRAM +++++++++++++++++++++++++++++++++++++++++++++++++++++
870                                         !
880                                         !
890    CALL Init(Vmsr,Vmsn,Vmst,R,Etgo,At,A,Ka,Aa,Gpre)
900                                         ! INPUT AND CALCULATE REQUIRED
910                                         ! PARAMETERS.
920                                         !
930    CALL Bcksctr(Ka,Nbs)                 ! CALCULATE THE NBCS.
940    PRINT "NBS=";Nbs
950                                         !
960    Atgt=PI*A*A
970    PRINT "ATGT=";Atgt
980    Sasph=4*PI*R*R
990    Prat=(Vmsr-Vmsn)/(Gpre*Gpre*Vmst)
1000   Er=(Prat)*(Sasph*Sasph/Etgo)*(EXP(2*At*R)/(Aa*Atgt))/Nbs
1010                                        !
1020   PRINT "Er=";Er
1030   PRINT "EtGo=";Etgo
1040   PRINT "Go=";Etgo/Er
1050   PRINT "EtEr=";Er*Er
1060                                        !
1070                                        !
1080   !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1090   !
1100   !++++ MAIN PROGRAM CONCLUSION ++++++++++++++++++++++++++++++++++++++++++
1110                                        !
1120   PRINT CHR$(12)
1130   PRINTER IS 1
1140   PRINT
1150   PRINT "END"                          ! ADVISES THE USER OF THE MAIN
1160   PRINT                                ! PROGRAM'S CONCLUSION.
1170   END
1180                                        !
1190   !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1200   !
1210   !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
1220   !
1230   !
```

```
1240  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
1250  SUB Init(Vmsr,Vmsn,Vmst,R,Etgo,At,A,Ka,Aa,Gpre)
1260  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1270  REM   THIS MODULE DETERMINES THE VALUES OF THE PARAMETERS REQUIRED FOR
1280  REM   THE CALCULATION OF Er.
1290  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1300  !
1310  !++++ MODULE VARIABLE DECLARATIONS AND DEFINITIONS +++++++++++++++++++++
1320                                       !
1330                                       !
1340  REAL Ch                              ! A CONTROL VARIABLE USED FOR
1350                                       ! CONTROLLING SELECTION OF THE
1360                                       ! CORRECT GAIN DATA SETS.
1370                                       !
1380  REAL Vr(60)                          ! THE ARRAY REPRESENTING THE
1390                                       ! MEASURED TARGET RETURN VOLTAGES.
1400                                       !
1410  REAL J,Jt,Jn,Jtn                     ! THE NUMBER OF ARRAY ELEMENTS MINUS
1420                                       ! ONE AND A LOOP CONTROL VARIABLE.
1430                                       !
1440  REAL Tc                              ! THE AMBIENT TEMPERATURE IN DEG. C.
1450                                       !
1460  REAL Patm                            ! THE ATMOSPHERIC PRESSURE MEASURED
1470                                       ! IN MILLIBARS.
1480                                       !
1490  REAL Rh                              ! THE RELATIVE HUMIDITY MEASURED IN
1500                                       ! PERCENT.
1510                                       !
1520  REAL Tgttm                           ! THE MEASURED TIME FOR THE ACOUSTIC
1530                                       ! SIGNAL TO MAKE THE ROUND TRIP
1540                                       ! BETWEEN THE ARRAY AND THE TARGET.
1550                                       !
1560  REAL Dia                             ! THE MEASURED DIAMETER OF THE
1570                                       ! TARGET SPHERE.
1580                                       !
1590  REAL Nuspkr                          ! THE NUMBER OF SPEAKERS IN THE
1600                                       ! ARRAY.
1610                                       !
1620  REAL Diaspkr                         ! THE AVERAGE DIAMETER OF AN ARRAY
1630                                       ! SPEAKER.
1640                                       !
1650  REAL Vt(60)                          ! THE ARRAY REPRESENTING THE
1660                                       ! MEASURED TRANSMISSION VOLTAGES.
1670                                       !
1680  REAL Noise(60)                       ! THE ARRAY REPRESENTING THE
1690                                       ! MEASURED NOISE RETURN VOLTAGES.
1700                                       !
1710  REAL Tnoise(60)                      ! THE ARRAY REPRESENTING THE
1720                                       ! MEASURED TRANSMISSION VOLTAGES FOR
1730                                       ! NOISE DETERMINATION.
1740                                       !
1750  REAL Tcn                             ! Tc FOR NOISE DETERMINATION.
1760                                       !
1770  REAL Rhn                             ! Rh FOR NOISE DETERMINATION.
1780                                       !
```

```
1790  REAL Patmn                          ! Patm FOR NOISE DETERMINATION.
1800                                      !
1810  REAL Efreqn                         ! Efreqn=Efreq.
1820                                      !
1830  REAL Cn                             ! C FOR NOISE DETERMINATION.
1840                                      !
1850  REAL Atn                            ! At FOR NOISE DETERMINATION.
1860                                      !
1870  REAL Vmstn                          ! Vmst FOR NOISE DETERMINATION.
1880                                      !
1890  REAL Rspkr                          ! THE AVERAGE RADIUS OF AN ARRAY
1900                                      ! SPEAKER.
1910                                      !
1920                                      !
1930  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
1940  !
1950  !++++ DETERMINING SYSTEM PARAMETERS +++++++++++++++++++++++++++++++++++++
1960                                      !
1970                                      !
1980  PRINTER IS 1
1990  PRINT "WHICH TARGET DATA SET ";
2000  PRINT "DO YOU WISH ANALYZED?"
2010  PRINTER IS 701
2020  INPUT Sphere
2030  PRINT "TARGET NO. ";Sphere
2040  PRINT
2050                                      !
2060  IF Sphere=1 THEN
2070     Ch=3.0
2080     CALL Gaino(Ch,Etgo)
2090     CALL Sphere1(Vr(*),J,Tc,Patm,Rh,Tgttm,Dia,Efreq,Nuspkr,Diaspkr,Gpre)
2100     CALL Trans30(Jt,Vt(*))
2110     CALL Noise30(Jn,Jtn,Noise(*),Tnoise(*),Tcn,Rhn,Patmn,Efreqn)
2120  END IF
2130                                      !
2140  IF Sphere=2 THEN
2150     Ch=3.5
2160     CALL Gaino(Ch,Etgo)
2170     CALL Sphere2(Vr(*),J,Tc,Patm,Rh,Tgttm,Dia,Efreq,Nuspkr,Diaspkr,Gpre)
2180     CALL Trans35(Jt,Vt(*))
2190     CALL Noise35(Jn,Jtn,Noise(*),Tnoise(*),Tcn,Rhn,Patmn,Efreqn)
2200  END IF
2210                                      !
2220  IF Sphere=3 THEN
2230     Ch=3.5
2240     CALL Gaino(Ch,Etgo)
2250     CALL Sphere3(Vr(*),J,Tc,Patm,Rh,Tgttm,Dia,Efreq,Nuspkr,Diaspkr,Gpre)
2260     CALL Trans35(Jt,Vt(*))
2270     CALL Noise35(Jn,Jtn,Noise(*),Tnoise(*),Tcn,Rhn,Patmn,Efreqn)
2280  END IF
2290                                      !
```

```
2300  IF Sphere=4 THEN
2310     Ch=3.5
2320     CALL Gaino(Ch,Etgo)
2330     CALL Sphere4(Vr(*),J,Tc,Patm,Rh,Tgttm,Dia,Efreq,Nuspkr,Diaspkr,Gpre)
2340     CALL Trans35(Jt,Vt(*))
2350     CALL Noise35(Jn,Jtn,Noise(*),Tnoise(*),Tcn,Rhn,Patmn,Efreqn)
2360  END IF
2370                                      !
2380  CALL Voltsms(Vr(*),J,Vmsr)
2390  PRINT "VMSR=";Vmsr
2400  CALL Atten(Tcn,Rhn,Patmn,Cn,Atn,Efreqn)
2410  CALL Voltsms(Tnoise(*),Jtn,Vmstn)
2420  CALL Voltsms(Noise(*),Jn,Vmsn)
2430  CALL Voltsms(Vt(*),Jt,Vmst)
2440  PRINT "VMST=";Vmst
2450                                      !
2460  A=Dia/2
2470  PRINT "A=";A
2480  CALL Range(Tgttm,A,Tc,Rh,Patm,C,At,Efreq,R)
2490  PRINT "C=";C
2500  PRINT "AT=";At
2510  PRINT "ATTEN=";EXP(At*R)
2520  PRINT "R=";R
2530  Vmsn=Vmsn*EXP(2*R*(Atn-At))*Vmst/Vmstn
2540  PRINT "VMSN=";Vmsn
2550  Ka=2*PI*Efreq*A/C
2560  PRINT "KA=";Ka
2570  Rspkr=Diaspkr/2
2580  Aa=Nuspkr*PI*Rspkr*Rspkr
2590  PRINT "AA=";Aa
2600                                      !
2610                                      !
2620  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
2630  !
2640  !++++ MODULE CONCLUSION +++++++++++++++++++++++++++++++++++++++++++++++
2650                                      !
2660  SUBEND
2670                                      !
2680  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
2690  !
2700  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
2710  !
```

```
2720  !
2730  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
2740  SUB Gaino(Ch,Etgo)
2750  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
2760  REM   THIS MODULE DETERMINES THE VALUE OF EtGo.
2770  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
2780  !
2790  !++++ MODULE VARIABLE DECLARATIONS AND DEFINITIONS ++++++++++++++++++++++
2800                                       !
2810                                       !
2820  REAL Vrms                            ! ROOT MEAN SQUARE VOLTAGE MEASURED
2830                                       ! IN GAIN DETERMINATION.
2840                                       !
2850  REAL Ir                              ! MEASURED INTENSITY AT R;
2860                                       ! Ptgt/Atgt.
2870                                       !
2880  REAL Za                              ! ACOUSTIC ARRAY IMPEDANCE.
2890                                       !
2900                                       !
2910  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
2920  !
2930  !++++ DETERMINATION OF EtGo +++++++++++++++++++++++++++++++++++++++++++++
2940                                       !
2950                                       !
2960  IF Ch=3.0 THEN CALL Gain30(Vrms,Tc,Rh,R,Patm,Ir,Efreq,Za)
2970  IF Ch=3.5 THEN CALL Gain35(Vrms,Tc,Rh,R,Patm,Ir,Efreq,Za)
2980                                       ! OBTAINING VALUES FROM APPROPRIATE
2990                                       ! DATA.
3000                                       !
3010  CALL Atten(Tc,Rh,Patm,C,At,Efreq)    ! CALCULATING AT.
3020  Pt=Vrms*Vrms/Za
3030  Etgo=Ir*4*PI*R*R*EXP(At*R)/Pt        ! CALCULATING EtGo.
3040                                       !
3050                                       !
3060  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3070  !
3080  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
3090                                       !
3100  SUBEND
3110                                       !
3120  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3130  !
3140  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
3150  !
```

```
3160  !
3170  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
3180  SUB Voltsms(Volt(*),J,Vms)
3190  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3200  REM   THIS MODULE DETERMINES THE MEAN SQUARE VALUE OF THE INPUT VOLTAGE
3210  REM   ARRAY.
3220  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3230  !
3240  !++++ MODULE VARIABLE DECLARATIONS AND DEFINITIONS +++++++++++++++++++++
3250                                      !
3260                                      !
3270  !REAL VOLT(*)                       ! THE ARRAY REPRESENTING THE INPUT
3280                                      ! VOLTAGE ARRAY.
3290                                      !
3300  REAL Volts                          ! THE SUM OF VOLT(*)'S ELEMENTS.
3310                                      !
3320  REAL Vavg                           ! THE AVERAGE VALUE OF VOLT(*).
3330                                      !
3340  REAL V                              ! THE DIFFERENCE BETWEEN VOLT(I)
3350                                      ! AND VAVG.
3360                                      !
3370  !REAL Vms                           ! THE MEAN SQUARE VALUE OF VOLT(*).
3380                                      !
3390                                      !
3400  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3410  !
3420  !++++ DETERMINATION OF VMS +++++++++++++++++++++++++++++++++++++++++++++
3430                                      !
3440                                      !
3450  Volts=0
3460  FOR I=0 TO J
3470    Volts=Volts+Volt(I)
3480  NEXT I
3490  Vavg=Volts/(J+1)
3500  FOR I=0 TO J
3510    V=Volt(I)-Vavg
3520    Vms=(V*V)+Vms
3530  NEXT I
3540  Vms=Vms/(J+1)
3550                                      !
3560                                      !
3570  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3580  !
3590  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
3600                                      !
3610  SUBEND
3620                                      !
3630  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3640  !
3650  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
3660  !
```

```
3670  !
3680  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
3690  SUB Range(Tgttm,A,Tc,Rh,Patm,C,At,Efreq,R)
3700  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3710  REM   THIS MODULE DETERMINES THE AVERAGE PATH ATTENUATION AND THE RANGE
3720  REM   BETWEEN THE ARRAY AND THE TARGET CENTER.
3730  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3740  !
3750  !++++ DETERMINATION OF R AND AT ++++++++++++++++++++++++++++++++++++++++
3760                                      !
3770                                      !
3780  Tgttm=Tgttm/1000
3790  CALL Atten(Tc,Rh,Patm,C,At,Efreq)   ! AT AND C DETERMINED.
3800  R=Tgttm*C/2+A                       ! R DETERMINED.
3810                                      !
3820                                      !
3830  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3840  !
3850  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
3860                                      !
3870  SUBEND
3880                                      !
3890  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3900  !
3910  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
3920  !
```

```
3930  !
3940  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
3950  SUB Atten(Tc,Rh,Patm,Cmoist,At,Efreq)
3960  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
3970  REM   THIS MODULE CALCULATES THE ATTENUATION USING RELATIVE HUMIDITY
3980  REM   IN PERCENT, ATMOSPHERIC PRESSURE IN MILLIBARS, AND TEMPERATURE
3990  REM   IN DEGREES CENTIGRADE.
4000  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4010  !
4020  !++++ MODULE VARIABLE DECLARATIONS AND DEFINITIONS ++++++++++++++++++++
4030                                      !
4040                                      !
4050  REAL Tk                             ! THE AMBIENT TEMPERATURE IN DEG. K.
4060                                      !
4070  REAL Es                             ! THE SATURATION VAPOR PRESSURE.
4080                                      !
4090  REAL Pratio                         ! THE RATIO OF WATER PRESSURE IN
4100                                      ! MILLIBARS TO ATMOSPHERIC PRESSURE
4110                                      ! IN MILLIBARS.
4120                                      !
4130  REAL Cdry                           ! THE SPEED OF SOUND IN DRY AIR.
4140                                      !
4150  REAL H                              !
4160                                      !
4170  REAL Past                           ! P ASTERISK.
4180                                      !
4190  REAL Tast                           ! T ASTERISK.
4200                                      !
4210  REAL Fm                             ! MAX. FREQ.
4220                                      !
4230  REAL Amax                           ! MAX EXPECTED ATTENUATION.
4240                                      !
4250  REAL Frat                           ! EFREQ/FM
4260                                      !
4270  REAL F2                             ! FRAT*FRAT
4280                                      !
4290  REAL Acl,Amol                       ! ATTENUATION EXPONENT COMPONENTS.
4300                                      !
4310                                      !
4320  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4330  !
```

```
4340  !++++ CALCULATING THE ATTENUATION ++++++++++++++++++++++++++++++++++++++
4350                                        !
4360                                        !
4370  Tk=Tc+273.15                          ! CONVERSION OF TEMP IN CELSIUS TO
4380                                        ! TEMP IN KELVIN.
4390                                        !
4400  Rh=Rh/100                             ! CONVERSION OF RH IN PCT. TO PRATI0.
4410  Es=10^(9.4-2353/Tk)                   !
4420  Pratio=Es*Rh/(Patm-Es*(1+Rh))         !
4430                                        !
4440  Cdry=20.05*SQR(Tk)                    ! CALCULATION OF THE SPEED OF SOUND.
4450  Cmoist=Cdry*(1+.14*Pratio)
4460                                        !
4470  H=Pratio*100
4480  Past=Patm/1014
4490  Tast=(1.8*Tc+492)/519
4500  Fm=(10+6600*H+44400*H*H)*Past/(Tast^.8)
4510                                        !
4520  Amax=.0078*Fm*(Tast^(-2.5))*EXP(7.77*(1-1/Tast))
4530                                        !
4540  Frat=Efreq/Fm
4550  F2=Frat*Frat
4560                                        !
4570  Amol=Amax*SQR(.18*.18*F2+(2*F2/(1+F2))^2)/304.8
4580  Acl=1.74*10^(-10)*Efreq*Efreq
4590  At=(Amol+Acl)*LOG(10)/10
4600                                        !
4610                                        !
4620  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4630  !
4640  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
4650                                        !
4660  SUBEND
4670                                        !
4680  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4690  !
4700  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
4710  !
```

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4720  !
4730  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
4740  SUB Bcksctr(Ka,Nbs)
4750  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4760  REM   THIS MODULE CALCULATES THE BACKSCATTERED CROSS SECTION NORMALIZED
4770  REM   BY THE CROSS SECTIONAL AREA OF THE SPHERE.
4780  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
4790  !
4800  !++++ MODULE VARIABLE DECLARATIONS AND DEFINITIONS +++++++++++++++++++++
4810                                       !
4820                                       !
4830  REAL L                               ! ORDER OF THE FUNCTION CALCULATED
4840                                       ! IN A SUBROUTINE.  IT IS USED
4850                                       ! AS A LOOP INDEX AND TO DESIGNATE
4860                                       ! THE ARRAY ELEMENT FOR THE FUNCTION
4870                                       ! OF CORRESPONDING ORDER.
4880                                       !
4890  REAL Lmax                            ! MAX. SIGNIFICANT ORDER OF THE SUM
4900                                       ! DETERMINING THE NORMALIZED
4910                                       ! BACKSCATTERING CROSS SECTION
4920                                       ! (NBCS).
4930                                       !
4940  REAL J(51)                           ! ARRAY FOR THE REGULAR SPHERICAL
4950                                       ! BESSEL FUNCTIONS (RSB) OF
4960                                       ! ARGUMENT KA AND ORDER 0 THROUGH
4970                                       ! LMAX.
4980                                       !
4990  REAL Dj(51)                          ! ARRAY FOR THE DERIVATIVES OF THE
5000                                       ! RSB (THE DRSB).
5010                                       !
5020  REAL Y(51)                           ! ARRAY FOR THE IRREGULAR SPHERICAL
5030                                       ! BESSEL FUNCTIONS (ISB) OF
5040                                       ! ARGUMENT KA AND ORDER 0 THROUGH
5050                                       ! LMAX.
5060                                       !
5070  REAL Dy(51)                          ! ARRAY FOR THE DERIVATIVES OF THE
5080                                       ! ISB (THE DISB).
5090                                       !
5100  REAL P(51)                           ! ARRAY FOR THE LEGENDRE
5110                                       ! POLYNOMIALS OF ARGUMENT COS(180).
5120                                       !
5130  REAL Fac                             ! FACTOR COMMON TO RSM AND ISM.
5140                                       !
5150  REAL Rsm                             ! THE REAL COMPONENT OF THE
5160                                       ! SCATTERING MODULUS, SM.
5170                                       !
5180  REAL Ism                             ! THE IMAGINARY COMPONENT OF THE
5190                                       ! SM.
5200                                       !
```

```
5210                                            ! ++++ NOTE ++++
5220                                            ! THE ARRAY DIMENSIONS (30+21) NEED
5230                                            ! TO BE INCREASED TO ACCOMODATE
5240                                            ! KA>30.
5250                                            !
5260                                            !
5270  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5280  !
5290  !++++ CALCULATING THE NBCS +++++++++++++++++++++++++++++++++++++++++++++++
5300                                            !
5310  Lmax=INT(Ka+21)
5320                                            !
5330  CALL Drsb(Lmax,Ka,Dj(*),J(*))             ! CALCULATE THE DERIVATIVES OF THE
5340                                            ! REGULAR SPHERICAL BESSEL FUNCTIONS
5350                                            ! OF ORDER 0 THROUGH LMAX.
5360  CALL Disb(Lmax,Ka,Dy(*),Y(*))             ! CALCULATE THE DERIVATIVES OF THE
5370                                            ! IRREGULAR SPHERICAL BESSEL FUNC'NS
5380                                            ! OF ORDER 0 THROUGH LMAX.
5390                                            !
5400  CALL Leg180(Lmax,Ka,P(*))                 ! CALCULATE THE ARRAY P(*).
5410                                            !
5420  Rsm=0                                     ! CALCULATING THE REAL AND IMAGINARY
5430  Ism=0                                     ! COMPONENTS OF THE SM.
5440  FOR L=0 TO Lmax
5450    Fac=(Dj(L)*(2*L+1)*P(L))/(Dj(L)*Dj(L)+Dy(L)*Dy(L))
5460    Rsm=Rsm+(Fac*Dy(L))
5470    Ism=Ism+(Fac*Dj(L))
5480  NEXT L
5490                                            !
5500  Nbs=4*(Rsm*Rsm+Ism*Ism)/(Ka*Ka)           ! CALCULATING THE NBCS.
5510                                            !
5520                                            !
5530  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5540  !
5550  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++++
5560                                            !
5570  SUBEND
5580                                            !
5590  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5600  !
5610  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
5620  !
```

```
5630  !
5640  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
5650  SUB Drsb(Lmax,Ka,Dj(*),J(*))
5660  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5670  REM   THIS MODULE CALCULATES THE DERIVATIVES OF THE REGULAR SPHERICAL
5680  REM   BESSEL FUNCTIONS (DRSB) OF ARGUMENT KA AND ORDERS 0 THROUGH LMAX.
5690  REM   THE CALCULATION OF THE DERIVATIVE INVOLVES THE RSB'S OF THE SAME
5700  REM   AND PREVIOUS ORDER.
5710  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5720  !
5730  !++++ CALCULATING THE DERIVATIVES OF THE RSB +++++++++++++++++++++++++++
5740                                      !
5750                                      !
5760  CALL Rsb(Lmax,Ka,J(*))              ! CALCULATE THE RSB VALUES J(*).
5770                                      !
5780  RAD                                 ! ANGLE UNITS FOR DRSB CALC'NS.
5790  Dj(0)=(-J(0)+COS(Ka))/Ka            ! CALCULATE THE INITIAL DRSB DJ(0).
5800  FOR L=1 TO Lmax                     ! CALCULATE THE REMAINING DRSB.
5810    Dj(L)=-((L+1)/Ka)*J(L)+J(L-1)
5820  NEXT L
5830                                      !
5840                                      !
5850  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5860  !
5870  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
5880                                      !
5890  SUBEND
5900                                      !
5910  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5920  !
5930  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
5940  !
```

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5950  !
5960  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
5970  SUB Rsb(Lmax,Ka,J(*))
5980  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
5990  REM   THIS MODULE CALCULATES THE REGULAR SPHERICAL BESSEL FUNCTIONS
6000  REM   (RSB) OF ARGUMENT KA AND ORDERS 0 THROUGH LMAX.   THE
6010  REM   CALCULATION USES THE CONTINUED FRACTION APPROACH.
6020  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
6030  !
6040  !++++ MODULE VARIABLE DECLARATIONS AND DEFINITIONS +++++++++++++++++++++
6050                                      !
6060                                      !
6070  REAL Rl                             ! THE EVOLVING RATIO OF J(0)/J(L).
6080                                      !
6090  REAL Numr                           ! THE EVOLVING NUMERATOR FACTOR USED
6100                                      ! IN CALCULATION OF RL.
6110                                      !
6120  REAL Denr                           ! THE EVOLVING DENOMINATOR FACTOR
6130                                      ! USED IN CALCULATION OF RL.
6140                                      !
6150  REAL Scexp                          ! EVOLVING SCALING EXPONENT FOR VERY
6160                                      ! LARGE OR VERY SMALL VALUES OF RL.
6170                                      !
6180  REAL Nflag                          ! 0 FLAG FOR NUMR.
6190                                      !
6200  REAL Dflag                          ! 0 FLAG FOR DENR.
6210                                      !
6220  REAL Bnm                            ! EVOLVING TERM USED IN THE
6230                                      ! CALCULATION OF NUMR AND DENR.
6240                                      !
6250  REAL Binc                           ! INCREMENT USED IN THE EVOLUTION
6260                                      ! OF BNM.
6270                                      !
6280                                      !
6290  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
6300  !
6310  !++++ CALCULATING THE RSB ++++++++++++++++++++++++++++++++++++++++++++++
6320                                      !
6330                                      !
6340  RAD                                 ! ANGLE UNITS FOR CALC'NS.
6350                                      !
6360                                      ! *** CALCULATE THE INITIAL ***
6370  J(0)=(SIN(Ka))/Ka                   ! *** RSB, J(0).          ***
6380                                      !
```

```
6390                                      ! *** CALCULATING THE RATIO ***
6400  FOR L=1 TO Lmax                     ! *** J(0)/J(L).            ***
6410    Rl=1.0                            ! INITIALIZING RL.
6420    Numr=0.                           ! INITIALIZING NUMR.
6430    Denr=0.                           ! INITIALIZING DENR.
6440    Scexp=0.                          ! INITIALIZING SCEXP.
6450    Nflag=0.                          ! INITIALIZING NFLAG.
6460    Dflag=0.                          ! INITIALIZING DFLAG.
6470    Bnm=1.0/Ka                        ! INITIALIZING BNM.
6480    Binc=2.0/Ka                       ! CALCULATE BINC.
6490                                      !
6500                                      ! * CALCULATE THE L "NAKED"     *
6510    FOR Ordcnt=1 TO L                 ! * NUMR FACTORS OF J(0)/J(L). *
6520      Bnm=Bnm+Binc                    ! INCREMENT BNM.
6530      IF Nflag=0 THEN                 ! CHECK NUMR=0 FLAG NOT SET;
6540        Numr=Bnm-Numr                 ! UPDATE NUMR IF NUMR<>0.
6550        IF Numr<>0 THEN
6560          Rl=Rl*Numr                  ! UPDATE RL IF UPDATED NUMR<>0.
6570          Numr=1.0/Numr               ! PREPARE NUMR FOR NEXT EVOLUTION.
6580                                      ! CHECK SCALING OF UPDATED RL.
6590          IF ABS(Rl)>10^250 THEN CALL Scale(Rl,Scexp)
6600        ELSE
6610          Nflag=1
6620        END IF
6630      ELSE                            ! IF NFLAG<>0 THE PREVIOUS STEPS ARE
6640        Nflag=0                       ! DEFERRED FOR ONE INCREMENT OF BNM.
6650      END IF
6660    NEXT Ordcnt
6670                                      ! * CALCULATE THE REMAINING NUMR *
6680                                      ! * AND DENR TERMS OF J(0)/J(L). *
6690                                      !   NOTE WHEN NUMR=DENR THEN
6700                                      !   RL=J(0)/J(L).
6710    WHILE Numr<>Denr OR Nflag=1 OR Dflag=1
6720      Bnm=Bnm+Binc                    ! INCREMENT BNM.
6730                                      !
6740      IF Nflag=0 THEN                 ! CHECK NUMR=0 FLAG NOT SET;
6750        Numr=Bnm-Numr                 ! UPDATE NUMR IF NUMR<>0.
6760        IF Numr<>0 THEN
6770          Rl=Rl*Numr                  ! UPDATE RL IF UPDATED NUMR<>0.
6780          Numr=1.0/Numr               ! PREPARE NUMR FOR NEXT EVOLUTION.
6790                                      ! CHECK SCALING OF UPDATED RL.
6800          IF ABS(Rl)>10^250 THEN CALL Scale(Rl,Scexp)
6810        ELSE
6820          Nflag=1
6830        END IF
6840      ELSE                            ! IF NFLAG<>0 THE PREVIOUS STEPS ARE
6850        Nflag=0                       ! DEFERRED FOR ONE INCREMENT OF BNM.
6860      END IF
6870                                      !
6880      IF Dflag=0 THEN                 ! CHECK DENR=0 FLAG NOT SET;
6890        Denr=Bnm-Denr                 ! UPDATE DENR IF DENR<>0.
6900        IF Denr<>0 THEN
6910          Rl=Rl/Denr                  ! UPDATE RL IF UPDATED DENR<>0.
6920          Denr=1.0/Denr               ! PREPARE DENR FOR NEXT EVOLUTION.
```

```
6930                                        ! CHECK SCALING OF UPDATED RL.
6940          IF ABS(Rl)>10^250 THEN CALL Scale(Rl,Scexp)
6950        ELSE
6960          Dflag=1
6970        END IF
6980      ELSE                              ! IF DFLAG<>0 THE PREVIOUS STEPS ARE
6990        Dflag=0                         ! DEFERRED FOR ONE INCREMENT OF BNM.
7000      END IF
7010                                        !
7020    END WHILE
7030                                        !
7040                                        !
7050    J(L)=J(0)/(Rl*10^(Scexp))           ! *** J(L) DETERMINED. ***
7060                                        !
7070  NEXT L
7080                                        !
7090                                        !
7100  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7110  !
7120  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
7130                                        !
7140  SUBEND
7150                                        !
7160  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7170  !
7180  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
7190  !
```

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7200  !
7210  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
7220  SUB Scale(Rl,Scexp)
7230  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7240  REM   THIS MODULE SCALES RL TO MAINTAIN NUMERICAL ACCURACY IN THE
7250  REM   ONGOING CALCULATION OF RL.
7260  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7270  !
7280  !++++ SCALING RL +++++++++++++++++++++++++++++++++++++++++++++++++++++++
7290                                      !
7300                                      !
7310  IF Rl>10^250 THEN
7320    Rl=Rl*10^(-250)
7330    Scexp=Scexp+250
7340  END IF
7350  IF Rl<10^(-250) THEN
7360    Rl=Rl*10^250
7370    Scexp=Scexp-250
7380  END IF
7390                                      !
7400                                      !
7410  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7420  !
7430  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
7440                                      !
7450  SUBEND
7460                                      !
7470  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7480  !
7490  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
7500  !
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7510  !
7520  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
7530  SUB Disb(Lmax,Ka,Dy(*),Y(*))
7540  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7550  REM   THIS MODULE CALCULATES THE DERIVATIVES OF THE IRREGULAR SPHERICAL
7560  REM   BESSEL FUNCTIONS OF ARGUMENT KA AND ORDERS 0 THROUGH LMAX.  THE
7570  REM   CALCULATION OF THE DERIVATIVE USES THE ISB'S OF THE SAME AND
7580  REM   PREVIOUS ORDER.
7590  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7600  !
7610  !++++ CALCULATING THE DERIVATIVES OF THE ISB +++++++++++++++++++++++++++
7620                                      !
7630                                      !
7640  CALL Isb(Lmax,Ka,Y(*))              ! CALCULATE THE ISB VALUES.
7650                                      !
7660  RAD                                 ! ANGLE UNITS FOR CALC'NS.
7670  Dy(0)=(-Y(0)+SIN(Ka))/Ka            ! THE INITIAL ISB DERIVATIVE.
7680  FOR L=1 TO Lmax                     ! THE REMAINING ISB DERIVATIVES.
7690    Dy(L)=-((L+1)/Ka)*Y(L)+Y(L-1)
7700  NEXT L
7710                                      !
7720                                      !
7730  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7740  !
7750  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
7760                                      !
7770  SUBEND
7780                                      !
7790  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7800  !
7810  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
7820  !
```

```
7830  !
7840  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
7850  SUB Isb(Lmax,Ka,Y(*))
7860  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7870  REM   THIS MODULE CALCULATES THE IRREGULAR SPHERICAL BESSEL FUNCTIONS
7880  REM   OF ARGUMENT KA AND  ORDER L THROUGH LMAX.   THE CALCULATION IS
7890  REM   ACCOMPLISHED BY FORWARD RECURSION.
7900  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
7910  !
7920  !++++ CALCULATING THE ISB +++++++++++++++++++++++++++++++++++++++++++++
7930                                      !
7940                                      !
7950  RAD                                 ! ANGLE UNITS FOR CALC'NS.
7960  Y(0)=-COS(Ka)/Ka                    ! INITIAL ISB'S CALCULATED.
7970  Y(1)=(Y(0)-SIN(Ka))/Ka
7980  FOR L=2 TO Lmax                     ! REMAINING ISB'S CALC'D.
7990    Y(L)=((2*L-1)/Ka)*Y(L-1)-Y(L-2)
8000  NEXT L
8010                                      !
8020                                      !
8030  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8040  !
8050  !++++ MODULE CONCLUSION +++++++++++++++++++++++++++++++++++++++++++++++
8060                                      !
8070  SUBEND
8080                                      !
8090  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8100  !
8110  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
8120  !
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8130  !
8140  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
8150  SUB Leg180(Lmax,Ka,P(*))
8160  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8170  REM   THIS MODULE CALCULATES THE LEGENDRE POLYNOMIALS OF ARGUMENT -1 AND
8180  REM   ORDER 0 THROUGH LMAX.  CALCULATION IS ACCOMPLISHED BY FORWARD
8190  REM   RECURSION.
8200  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8210  !
8220  !++++ CALCULATING THE LEGENDRE POLYNOMIALS +++++++++++++++++++++++++++++
8230                                      !
8240  X=-1                                ! ARGUMENT OF THE LEG. POL'S.
8250                                      !
8260  P(0)=1                              ! INITIAL LEG. POL'S. CALC'D.
8270  P(1)=X
8280  FOR L=2 TO Lmax                     ! REMAINING LEG. POL'S. CALC'D.
8290    P(L)=(2-(1/L))*X*P(L-1)-(1-(1/L))*P(L-2)
8300  NEXT L
8310                                      !
8320  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8330  !
8340  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
8350                                      !
8360  SUBEND
8370                                      !
8380  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8390  !
8400  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
8410  !
```

```
8420  !
8430  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
8440  SUB Gain30(Vrms,Tc,Rh,R,Patm,Ir,Efreq,Za)
8450  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8460  REM   THIS MODULE CONTAINS THE DATA USED FOR DETERMINATION OF EtGo WITH
8470  REM   3.0V INPUT FROM THE POWER SUPPLY.
8480  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8490  !
8500  !++++ LOADING DATA +++++++++++++++++++++++++++++++++++++++++++++++++++++
8510                                      !
8520                                      !
8530  RESTORE G30
8540  READ Vrms,Tc,Rh,R,Patm,Ir,Efreq,Za
8550 G30:   DATA .426,21.0,50.0,5.30,1021.5,2.951E-3,5000,15.8
8560                                      !
8570                                      !
8580  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8590  !
8600  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
8610                                      !
8620  SUBEND
8630                                      !
8640  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8650  !
8660  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
8670  !
```

```
8680  !
8690  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
8700  SUB Gain35(Vrms,Tc,Rh,R,Patm,Ir,Efreq,Za)
8710  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8720  REM   THIS MODULE CONTAINS THE DATA USED FOR DETERMINATION OF EtGo WITH
8730  REM   3.5V INPUT FROM THE POWER SUPPLY.
8740  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8750  !
8760  !++++ LOADING DATA ++++++++++++++++++++++++++++++++++++++++++++++++++++++
8770                                      !
8780                                      !
8790  RESTORE G35
8800  READ Vrms,Tc,Rh,R,Patm,Ir,Efreq,Za
8810 G35:   DATA .523,21.0,50.0,5.30,1021.5,4.467E-3,5000,15.8
8820                                      !
8830                                      !
8840  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8850  !
8860  !++++ MODULE CONCLUSION +++++++++++++++++++++++++++++++++++++++++++++++++
8870                                      !
8880  SUBEND
8890                                      !
8900  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8910  !
8920  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
8930  !
```

```
8940  !
8950  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
8960  SUB Sphere1(Dt(*),J,Tc,Patm,Rh,Tgttm,Dia,Efreq,Nuspkr,Diaspkr,Gpre)
8970  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
8980  REM   THIS MODULE CONTAINS THE DATA USED FOR DETERMINATION OF Er WITH
8990  REM   TARGET NO. 1 EMPLOYED.
9000  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
9010  !
9020  !++++ LOADING DATA +++++++++++++++++++++++++++++++++++++++++++++++++++++
9030                                      !
9040                                      !
9050  J=39
9060  RESTORE S1
9070  FOR I=0 TO J
9080    READ Dt(I)
9090  NEXT I
9100  READ Tc,Patm,Rh,Tgttm,Dia,Efreq,Nuspkr,Diaspkr,Gpre
9110 S1:  DATA 9.54,9.4,8.32,6.42,3.88,.96,-2.07,-4.9,-7.24,-8.87
9120      DATA -9.64,-9.45,-8.32,-6.41,-3.83,-.89,2.14,4.94,7.26,8.88
9130      DATA 9.6,9.38,8.22,6.26,3.67,.73,-2.3,-5.11,-7.4,-8.96
9140      DATA -9.64,-9.39,-8.2,-6.21,-3.61,-.65,2.36,5.12,7.38,8.91
9150      DATA 20.2,1016.8,52.8,29.91,.2546,5000,19,.0762,11094
9160                                      !
9170                                      !
9180  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
9190  !
9200  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
9210                                      !
9220  SUBEND
9230                                      !
9240  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
9250  !
9260  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
9270  !
```

```
9280  !
9290  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
9300  SUB Sphere2(Dt(*),J,Tc,Patm,Rh,Tgttm,Dia,Efreq,Nuspkr,Diaspkr,Gpre)
9310  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
9320  REM   THIS MODULE CONTAINS THE DATA USED FOR DETERMINATION OF Er WITH
9330  REM   TARGET NO. 2 EMPLOYED.
9340  REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
9350  !
9360  !++++ LOADING DATA +++++++++++++++++++++++++++++++++++++++++++++++++++++
9370                                       !
9380                                       !
9390  J=39
9400  RESTORE S2
9410  FOR I=0 TO J
9420    READ Dt(I)
9430  NEXT I
9440  READ Tc,Patm,Rh,Tgttm,Dia,Efreq,Nuspkr,Diaspkr,Gpre
9450 S2:  DATA 4.72,4.66,4.16,3.23,2.01,.560,-.940,-2.35,-3.52,-4.36
9460      DATA -4.77,-4.70,-4.18,-3.25,-1.99,-.560,.960,2.34,3.51,4.34
9470      DATA 4.74,4.66,4.13,3.19,1.95,.500,-1.00,-2.40,-3.57,-4.38
9480      DATA -4.78,-4.68,-4.15,-3.20,-1.92,-.480,1.02,2.39,3.54,4.34
9490      DATA 21.0,1009.4,49.8,30.55,.1013,5000,19,.0762,11094
9500                                       !
9510                                       !
9520  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
9530  !
9540  !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
9550                                       !
9560  SUBEND
9570                                       !
9580  !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
9590  !
9600  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
9610  !
```

```
9620  !
9630  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
9640  SUB Sphere3(Dt(*),J,Tc,Patm,Rh,Tgttm,Dia,Efreq,Nuspkr,Diaspkr,Gpre)
9650  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
9660  REM   THIS MODULE CONTAINS THE DATA USED FOR DETERMINATION OF Er WITH
9670  REM   TARGET NO. 3 EMPLOYED.
9680  REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
9690  !
9700  !++++ LOADING DATA ++++++++++++++++++++++++++++++++++++++++++++++++++++++
9710                                       !
9720                                       !
9730  J=39
9740  RESTORE S3
9750  FOR I=0 TO J
9760    READ Dt(I)
9770  NEXT I
9780  READ Tc,Patm,Rh,Tgttm,Dia,Efreq,Nuspkr,Diaspkr,Gpre
9790 S3:  DATA 3.61,3.40,2.86,2.04,1.01,-.120,-1.25,-2.25,-3.04,-3.54
9800      DATA -3.68,-3.46,-2.91,-2.07,-1.04,.110,1.25,2.24,3.00,3.48
9810      DATA 3.63,3.39,2.84,2.02,.970,-.180,-1.30,-2.31,-3.08,-3.56
9820      DATA -3.69,-3.46,-2.89,-2.04,-.990,.160,1.29,2.26,3.05,3.50
9830      DATA 21.3,1009.6,49.8,30.59,.07634,5000,19,.0762,11094
9840                                       !
9850                                       !
9860  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
9870  !
9880  !++++ MODULE CONCLUSION +++++++++++++++++++++++++++++++++++++++++++++++++
9890                                       !
9900  SUBEND
9910                                       !
9920  !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
9930  !
9940  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
9950  !
```

```
9960  !
9970  !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
9980  SUB Sphere4(Dt(*),J,Tc,Patm,Rh,Tgttm,Dia,Efreq,Nuspkr,Diaspkr,Gpre)
9990  REM    ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
10000 REM    THIS MODULE CONTAINS THE DATA USED FOR DETERMINATION OF Er WITH
10010 REM    TARGET NO. 4 EMPLOYED.
10020 REM    ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
10030 !
10040 !++++ LOADING DATA +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
10050                                      !
10060                                      !
10070 J=39
10080 RESTORE S4
10090 FOR I=0 TO J
10100   READ Dt(I)
10110 NEXT I
10120 READ Tc,Patm,Rh,Tgttm,Dia,Efreq,Nuspkr,Diaspkr,Gpre
10130 S4: DATA 2.28,2.10,1.68,1.10,.420,-.320,-1.03,-1.64,-2.07,-2.31
10140     DATA -2.34,-2.12,-1.71,-1.12,-.440,.320,1.02,1.62,2.06,2.28
10150     DATA 2.29,2.10,1.68,1.09,.400,-.340,-1.04,-1.66,-2.10,-2.32
10160     DATA -2.35,-2.12,-1.70,-1.11,-.400,.340,1.04,1.64,2.08,2.28
10170     DATA 21.4,1009.9,49.6,30.64,.06240,5000,19,.0762,11094
10180                                      !
10190                                      !
10200 !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
10210 !
10220 !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++++++++
10230                                      !
10240 SUBEND
10250                                      !
10260 !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
10270 !
10280 !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
10290 !
```

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10300 !
10310 !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
10320 SUB Trans30(J,Dt(*))
10330 REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
10340 REM   THIS MODULE CONTAINS DATA USED IN DETERMINATION OF Er WITH
10350 REM   3.0V APPLIED.
10360 REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
10370 !
10380 !++++ LOADING DATA +++++++++++++++++++++++++++++++++++++++++++++++++++++
10390                                     !
10400                                     !
10410 J=39
10420 RESTORE T30
10430 FOR I=0 TO J
10440    READ Dt(I)
10450 NEXT I
10460 T30: DATA .690,.680,.590,.440,.320,.160,-.020,-.230,-.410,-.550
10470      DATA -.610,-.610,-.520,-.360,-.240,-.080,.110,.320,.490,.630
10480      DATA .690,.680,.590,.430,.320,.150,-.040,-.240,-.420,-.560
10490      DATA -.620,-.600,-.510,-.380,-.240,-.070,.120,.330,.500,.640
10500                                     !
10510                                     !
10520 !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
10530 !
10540 !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
10550                                     !
10560 SUBEND
10570                                     !
10580 !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
10590 !
10600 !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
10610 !
```

```
10620 !
10630 !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
10640 SUB Trans35(J,Dt(*))
10650 REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
10660 REM   THIS MODULE CONTAINS DATA USED IN DETERMINATION OF Er WITH
10670 REM   3.5V APPLIED.
10680 REM   ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
10690 !
10700 !++++ LOADING DATA +++++++++++++++++++++++++++++++++++++++++++++++++++++
10710                                     !
10720                                     !
10730 J=39
10740 RESTORE T35
10750 FOR I=0 TO J
10760    READ Dt(I)
10770 NEXT I
10780 T35:  DATA .830,.770,.640,.490,.290,.080,-.180,-.400,-.610,-.730
10790       DATA -.770,-.720,-.580,-.430,-.240,-.020,.240,.460,.660,.780
10800       DATA .830,.770,.630,.470,.290,.060,-.190,-.420,-.600,-.720
10810       DATA -.770,-.710,-.570,-.420,-.230,-.010,.240,.480,.670,.790
10820                                     !
10830                                     !
10840 !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
10850 !
10860 !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
10870                                     !
10880 SUBEND
10890                                     !
10900 !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
10910 !
10920 !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
10930 !
```

```
10940 !
10950 !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
10960 SUB Noise30(J,Jt,Dt(*),Tdt(*),Tcn,Rhn,Patmn,Efreqn)
10970 REM    ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
10980 REM    THIS MODULE CONTAINS DATA USED IN DETERMINATION OF Er WITH
10990 REM    3.0V APPLIED.
11000 REM    ++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
11010 !
11020 !++++ LOADING DATA +++++++++++++++++++++++++++++++++++++++++++++++++++++
11030                                     !
11040                                     !
11050 J=59
11060 RESTORE N30
11070 FOR I=0 TO J
11080    READ Dt(I)
11090 NEXT I
11100 Jt=39
11110 RESTORE Tn30
11120 FOR I=0 TO Jt
11130    READ Tdt(I)
11140 NEXT I
11150 READ Tcn,Rhn,Patmn,Efreqn
11160 N30:  DATA 0,0,-.02,-.03,-.04,-.06,-.08,-.1,-.11,-.12
11170       DATA -.12,-.11,-.1,-.08,-.07,-.05,-.04,-.01,0,0
11180       DATA 0,0,-.02,-.03,-.05,-.07,-.08,-.1,-.11,-.12
11190       DATA -.12,-.12,-.11,-.08,-.07,-.04,-.03,-.02,0,0
11200       DATA .01,0,0,-.03,-.04,-.07,-.09,-.11,-.12,-.12
11210       DATA -.12,-.12,-.11,-.08,-.07,-.04,-.03,0,0,.01
11220 Tn30: DATA .69,.68,.59,.44,.32,.16,-.02,-.23,-.41,-.55
11230       DATA -.61,-.61,-.52,-.36,-.24,-.08,.11,.32,.49,.63
11240       DATA .69,.68,.59,.43,.32,.15,-.04,-.24,-.42,-.56
11250       DATA -.62,-.6,-.51,-.38,-.24,-.07,.12,.33,.5,.64
11260       DATA 20.8,51.7,1016.7,5000
11270                                     !
11280                                     !
11290 !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
11300 !
11310 !++++ MODULE CONCLUSION ++++++++++++++++++++++++++++++++++++++++++++++++
11320                                     !
11330 SUBEND
11340                                     !
11350 !+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
11360 !
11370 !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
11380 !
```

```
11390 !
11400 !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
11410 SUB Noise35(J,Jt,Dt(*),Tdt(*),Tcn,Rhn,Patmn,Efreqn)
11420 REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
11430 REM   THIS MODULE CONTAINS DATA USED IN DETERMINATION OF Er WITH
11440 REM   3.5V APPLIED.
11450 REM   +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
11460 !
11470 !++++ LOADING DATA ++++++++++++++++++++++++++++++++++++++++++++++++++++++
11480                                      !
11490                                      !
11500 J=39
11510 RESTORE N35
11520 FOR I=0 TO J
11530    READ Dt(I)
11540 NEXT I
11550 Jt=39
11560 RESTORE Tn35
11570 FOR I=0 TO Jt
11580    READ Tdt(I)
11590 NEXT I
11600 READ Tcn,Rhn,Patmn,Efreqn
11610 N35:  DATA -.06,-.08,-.08,-.08,-.07,-.06,-.04,-.02,0,.02
11620       DATA .05,.06,.06,.06,.04,.03,.02,0,-.03,-.05
11630       DATA -.07,-.08,-.09,-.08,-.08,-.06,-.04,-.02,.01,.03
11640       DATA .04,.07,.06,.07,.05,.04,.01,-.01,-.04,-.06
11650 Tn35: DATA .74,.83,.82,.72,.53,.38,.18,-.06,-.31,-.53
11660       DATA -.68,-.77,-.75,-.64,-.46,-.3,-.1,.14,.4,.6
11670       DATA .75,.83,.81,.7,.5,.36,.16,-.08,-.33,-.54
11680       DATA -.69,-.76,-.74,-.63,-.46,-.28,-.08,.16,.41,.62
11690       DATA 21.3,51.3,1016.6,5000
11700                                      !
11710                                      !
11720 !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
11730 !
11740 !++++ MODULE CONCLUSION +++++++++++++++++++++++++++++++++++++++++++++++++
11750                                      !
11760 SUBEND
11770                                      !
11780 !++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
11790 !
11800 !$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
11810 !
```

APPENDIX E

ACOUSTIC ARRAY CALIBRATION DATA

This appendix contains data used to determine the calibration product $E_r E_t$. Data units are contained in square brackets.

For listings of the same type of data the units and uncertainty are explicitly given for the first datum. Remaining data in the listing have the same units and uncertainty as the first datum.

1. TWO WAY PROPAGATION PATH DATA

Pre-amplifier gain G_e and array impedance Z were previously determined by Moxcey. G_e was determined to be 11,094. Z was determined to be 15.8 [Ω]. [Ref. 8]

The remaining data are presented in the following tables.

V_{supply} is the voltage supplied by the HP3314A function generator to the pre-amplifier (see Figure 8).

Table 5a: Two Way Propagation Path Data for Sphere One, Collected 30 November 1987; Run *2: Data Re-measured Because First Run V_r (with 3.5 [V] Supply) Was Clipped.

Diameter = 2a = .2546 ± 0.0006 [m]
T_c = 20.2 ± 0.1 [° c]
Rh = 52.8 ± 0.1 [%]
P = 1016.8 ± 0.2 [mb]
No. pulses = 20
V_{supply} = 3.0 [V]
t_r = 30.31 [mS] - 400 [µS] = 29.91 ± 0.1 [mS]

t	V_t	t	V_t
2.25 ± 0.01[mS]	690 ± 5 [mV]	2.45	690
2.26	680	2.46	680
2.27	590	2.47	590
2.28	440	2.48	430
2.29	320	2.49	320
2.30	160	2.50	150
2.31	- 20	2.51	- 40
2.32	-230	2.52	-240
2.33	-410	2.53	-420
2.34	-550	2.54	-560
2.35	-610	2.55	-620
2.36	-610	2.56	-600
2.37	-520	2.57	-510
2.38	-360	2.58	-380
2.39	-240	2.59	-240
2.40	- 80	2.60	- 70
2.41	110	2.61	120
2.42	320	2.62	330
2.43	490	2.63	500
2.44	630	2.64	640

Table 5b: Two Way Propagation Path Data for Sphere One Continued, Collected 30 November 1987; Run *2: Data Re-measured Because First Run V_r (with 3.5 [V] Supply) Was Clipped.

t	V_r	t	V_r
33.68 ±0.01[mS]	9.54 ± 0.03 [V]	33.88	9.60
33.69	9.40	33.89	9.38
33.70	8.32	33.90	8.22
33.71	6.42	33.91	6.26
33.72	3.88	33.92	3.67
33.73	0.960	33.93	0.730
33.74	-2.07	33.94	-2.30
33.75	-4.90	33.95	-5.11
33.76	-7.24	33.96	-7.40
33.77	-8.87	33.97	-8.96
33.78	-9.64	33.98	-9.64
33.79	-9.45	33.99	-9.39
33.80	-8.32	34.00	-8.20
33.81	-6.41	34.01	-6.21
33.82	-3.83	34.02	-3.61
33.83	-0.890	34.03	-0.650
33.84	2.14	34.04	2.36
33.85	4.94	34.05	5.12
33.86	7.26	34.06	7.38
33.87	8.88	34.07	8.91

Table 6a: Two Way Propagation Path Data for Sphere Two, Collected 28 November 1987.

Diameter = 2a = 10.13 ± 0.01 [cm]
T_c = 21.0 ± 0.1 [° c]
Rh = 49.8 ± 0.1 [%]
P = 1009.4 ± 0.2 [mb]
No. pulses = 20
V_{supply} = 3.5 [V]
t_r = 30.95 [mS] - 400 [μS] = 30.55 ± 0.1 [mS]

t	V_t	t	V_t
2.26 ± 0.01[mS]	830 ± 5 [mV]	2.46	830
2.27	770	2.47	770
2.28	640	2.48	630
2.29	490	2.49	470
2.30	290	2.50	290
2.31	80	2.51	60
2.32	-180	2.52	-190
2.33	-400	2.53	-420
2.34	-610	2.54	-600
2.35	-730	2.55	-720
2.36	-770	2.56	-770
2.37	-720	2.57	-710
2.38	-580	2.58	-570
2.39	-430	2.59	-420
2.40	-240	2.60	-230
2.41	- 20	2.61	- 10
2.42	240	2.62	240
2.43	460	2.63	480
2.44	660	2.64	670
2.45	780	2.65	790

Table 6b: Two Way Propagation Path Data for Sphere Two Continued, Collected 28 November 1987.

t	V_r	t	V_r
34.13 ±0.01[mS]	4.72 ± 0.01 [V]	34.33	4.74
34.14	4.66	34.34	4.66
34.15	4.16	34.35	4.13
34.16	3.23	34.36	3.19
34.17	2.01	34.37	1.95
34.18	0.560	34.38	0.500
34.19	-0.940	34.39	-1.00
34.20	-2.35	34.40	-2.40
34.21	-3.52	34.41	-3.57
34.22	-4.36	34.42	-4.38
34.23	-4.77	34.43	-4.78
34.24	-4.70	34.44	-4.68
34.25	-4.18	34.45	-4.15
34.26	-3.25	34.46	-3.20
34.27	-1.99	34.47	-1.92
34.28	-0.560	34.48	-0.480
34.29	0.960	34.49	1.02
34.30	2.34	34.50	2.39
34.31	3.51	34.51	3.54
34.32	4.34	34.52	4.34

Table 7: Two Way Propagation Path Data for Sphere Three, Collected 28 November 1987.

Diameter = 2a = 7.634 ± 0.003 [cm]
T_c = 21.3 ± 0.1 [° c]
Rh = 49.8 ± 0.1 [%]
P = 1009.6 ± 0.2 [mb]
No. pulses = 20
V_{supply} = 3.5 [V]
t_r = 30.99 [mS] - 400 [µS] = 30.59 ± 0.1 [mS]

V_t data is the same as sphere two's (see Table 6a).

t	V_r	t	V_r
33.98 ±0.01[mS]	3.61 ± 0.03 [V]	34.18	3.63
33.99	3.40	34.19	3.39
34.00	2.86	34.20	2.84
34.01	2.04	34.21	2.02
34.02	1.01	34.22	0.970
34.03	-0.120	34.23	-0.180
34.04	-1.25	34.24	-1.30
34.05	-2.25	34.25	-2.31
34.06	-3.04	34.26	-3.08
34.07	-3.54	34.27	-3.56
34.08	-3.68	34.28	-3.69
34.09	-3.46	34.29	-3.46
34.10	-2.91	34.30	-2.89
34.11	-2.07	34.31	-2.04
34.12	-1.04	34.32	-0.990
34.13	0.110	34.33	0.160
34.14	1.25	34.34	1.29
34.15	2.24	34.35	2.26
34.16	3.00	34.36	3.05
34.17	3.48	34.37	3.50

Table 8: Two Way Propagation Path Data for Sphere Four, Collected 28 November 1987.

Diameter = 2a = 6.240 ± 0.008 [cm]
T_c = 21.4 ± 0.1 [° c]
Rh = 49.6 ± 0.1 [%]
P = 1009.9 ± 0.2 [mb]
No. pulses = 20
V_{supply} = 3.5 [V]
t_r = 31.04 [mS] - 400 [μS] = 30.64 ± 0.1 [mS]

V_t data is the same as sphere two's (see Table 6a).

t	V_r	t	V_r
33.83 ±0.01[mS]	2.28 ± 0.03 [V]	34.03	2.29
33.84	2.10	34.04	2.10
33.85	1.68	34.05	1.68
33.86	1.10	34.06	1.09
33.87	0.420	34.07	0.400
33.88	-0.320	34.08	-0.340
33.89	-1.03	34.09	-1.04
33.90	-1.64	34.10	-1.66
33.91	-2.07	34.11	-2.10
33.92	-2.31	34.12	-2.32
33.93	-2.34	34.13	-2.35
33.94	-2.12	34.14	-2.12
33.95	-1.71	34.15	-1.70
33.96	-1.12	34.16	-1.11
33.97	-0.440	34.17	-0.400
33.98	0.320	34.18	0.340
33.99	1.02	34.19	1.04
34.00	1.62	34.20	1.64
34.01	2.06	34.21	2.08
34.02	2.28	34.22	2.28

Table 9: Noise Data for 3.0 [V] Supply Voltage; Collected 30 November 1987.

T_c = 20.8 ± 0.1 [° c] Rh = 51.7 ± 0.1 [%] P = 1016.7 ± 0.2 [mb]

V_{supply} = 3.0 [V] No. pulses = 20

V_t data is the same as sphere one's (see Table 5a).

t	V_n	t	V_n
33.73 ±0.01[mS]	00 ± 5 [mV]	34.03	-120
33.74	00	34.04	-120
33.75	- 20	34.05	-110
33.76	- 30	34.06	- 80
33.77	- 40	34.07	- 70
33.78	- 60	34.08	- 40
33.79	- 80	34.09	- 30
33.80	-100	34.10	- 20
33.81	-110	34.11	00
33.82	-120	34.12	00
33.83	-120	34.13	10
33.84	-110	34.14	00
33.85	-100	34.15	00
33.86	- 80	34.16	- 30
33.87	- 70	34.17	- 40
33.88	- 50	34.18	- 70
33.89	- 40	34.19	- 90
33.90	- 10	34.20	-110
33.91	00	34.21	-120
33.92	00	34.22	-120
33.93	00	34.23	-120
33.94	00	34.24	-120
33.95	- 20	34.25	-110
33.96	- 30	34.26	- 80
33.97	- 50	34.27	- 70
33.98	- 70	34.28	- 40
33.99	- 80	34.29	- 30
34.00	-100	34.30	00
34.01	-110	34.31	00
34.02	-120	34.32	10

Table 10a: Noise Data for 3.5 [V] Supply Voltage; Collected 30 November 1987.

T_c = 21.3 ± 0.1 [° c]
Rh = 51.3 ± 0.1 [%]
P = 1016.6 ± 0.2 [mb]
No. pulses = 20
V_{supply} = 3.5 [V]

t	V_t	t	V_t
2.24 ±0.01[mS]	740 ± 5 [mV]	2.44	750
2.25	830	2.45	830
2.26	820	2.46	810
2.27	720	2.47	700
2.28	530	2.48	500
2.29	380	2.49	360
2.30	180	2.50	160
2.31	- 60	2.51	- 80
2.32	-310	2.52	-330
2.33	-530	2.53	-540
2.34	-680	2.54	-690
2.35	-770	2.55	-760
2.36	-750	2.56	-740
2.37	-640	2.57	-630
2.38	-460	2.58	-460
2.39	-300	2.59	-280
2.40	-100	2.60	- 80
2.41	140	2.61	160
2.42	400	2.62	410
2.43	600	2.63	620

Table 10b: Noise Data for 3.5 [V] Supply Voltage Continued; Collected 30 November 1987.

t	V_n	t	V_n
33.80 ±0.01[mS]	- 60 ± 5 [mV]	34.00	- 70
33.81	- 80	34.01	- 80
33.82	- 80	34.02	- 90
33.83	- 80	34.03	- 80
33.84	- 70	34.04	- 80
33.85	- 60	34.05	- 60
33.86	- 40	34.06	- 40
33.87	- 20	34.07	- 20
33.88	00	34.08	10
33.89	20	34.09	30
33.90	50	34.10	40
33.91	60	34.11	70
33.92	60	34.12	60
33.93	60	34.13	70
33.94	40	34.14	50
33.95	30	34.15	40
33.96	20	34.16	10
33.97	00	34.17	- 10
33.98	- 30	34.18	- 40
33.99	- 50	34.19	- 60

2. ONE WAY PROPAGATION PATH DATA

One way propagation path data are presented in the following tables. T_c and Rh are estimated. The estimates were based on hygrothermographic comparisons and 28 and 30 November data.

Table 11: One Way Propagation Path Data for 3.0 [V], 5000 [Hz] Continuous Wave Supply from the HP3314A Function Generator; Collected 7 December 1987.

$(V_{rms})_t$ = 0.246 ± 0.001 [V]
$T_c \approx 21.0$ [° c]
Rh ≈ 50.0 [%]
P = 1021.5 ± 0.2 [mb]
R = 5.30 [m] (calculated from two way propagation path data for spheres two, three and four)

I_r (referenced to 10^{-12} [W/m^2])

94.7 ± 0.1 [dB]
94.5
94.7
95.0
94.8
95.1
94.2

Table 12: One Way Propagation Path Data for 3.5 [V], 5000 [Hz] Continuous Wave Supply from the HP3314A Function Generator; Collected 7 December 1987.

$(V_{rms})_t$ = 0.523 ± 0.001 [V]
T_c ≈ 21.0 [° c]
Rh ≈ 50.0 [%]
P = 1021.5 ± 0.2 [mb]
R = 5.30 [m] (calculated from two way propagation path data for spheres two, three and four)

I_r (referenced to 10^{-12} [W/m^2])

97.4 ± 0.1 [dB]
96.4
95.1
97.0
96.2
97.1
96.4
95.4
96.3
97.4

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